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ORBIT DETERMINATION OF HILDA AND THULE ASTEROIDS

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Abstract

In the present paper, general computational algorithm for the planar restricted circular three-body problem in rotating synodic system is given. The algorithm is applied for orbit determination of Hilda and Thule asteroids. The results are illustrated graphically.

Key Words: Resonance, Hilda Asteroid, Thule Asteroid, planar restricted circular three-body problem

1. Introduction

The giant planet has captures the asteroid Hilda and about twenty smaller companions forming the Hilda group, and with Thule, which seems to be alone. The coupling of the orbit of Jupiter and the asteroid is given by the ratio of the period of the Jupiter to the period of asteroid P_J/P_A. In these cases this ratio may be written as a ratio of two small integers. For Hilda it is 3:2 and for Thule it is 4:3. Similar situations are sometimes found between two members of the satellite systems of large planets (with the large planet acting as first primary).

The early interest in resonances stemmed from the ability to deduce fairly accurate masses of the objects involved. In the last 19th century, for example, Simon Newcomb used perturbations in the orbit of the asteroid Polyhymnia to find the mass of Jupiter to be 1/1047.300 that of the Sun. This is nearly identical to the modern value of 1/1047.335 .

Hilda asteroids

Consists of asteroids with a semi-major axis between 3.7 AU and 4.2 AU, an eccentricity less than 0.3, and an inclination less than 20°. They do not form a true asteroid family, in the sense that they do not descend from a common parent object.

Instead, this is a *dynamical group* of bodies, made up of asteroids which, as said before, in a 3:2 orbital resonance with Jupiter.

One of the goals of theorists is to formulate a theory of the origin of the resonances, and why some generate gaps and others concentrations of asteroids. Cunningham, (1988) has found a difference in the local topology between the 2:1 Hecuba gap and 3:2 Hilda group based purely on gravity. As Cunningham, (1988) pointed out that, it appears there is a protection mechanism in the asteroid motion that permits clustering at some resonances and gaps at others.

The phase angle σ is defined to be

$$\sigma = Q\lambda - P\lambda_J + (P-Q)\varpi \ ,$$

where Q and P are integers, λ and λ_J are the mean longitudes of the asteroid and Jupiter, and ϖ is the longitude of perihelion of the asteroid (Chapman et al., 1978).

The stability of a resonance is determined by the phase angle σ. If it librates about some angle, usually 0 to 180°, the resulting resonances will be very stable. Such as the case for 4:3 (as Thule), 3:2 (as Hilda) and 2:1 (as Griqua).

Hildas move in their elliptical orbits so that their aphelia put them opposite Jupiter, or 60 degrees ahead of or behind Jupiter at the Lagrangian points (L_4 and L_5). The namesake is 153 Hilda, discovered by Johann Palisa in 1875. There are more than 1,100 known Hilda asteroids including unnumbered objects.

Hildas' surface colors often correspond to the low-albedo D-type and P-type, however, a small portion are C-type. The surface color of D-type and P-type asteroids such as Hildas and Trojans found in the outer main asteroid belt, are similar to cometary nuclei, and thus have similar mineralogical surfaces to cometary nuclei. This implies that they share a common origin.

The stability of the orbit of the asteroid is greatest when it passes through its perihelion just at the time when it-crosses the radius vector of Jupiter. So, at that moment the Sun, Hilda, and Jupiter are on one straight line, and Hilda itself happens to be at its closest distance to the Sun. We will call this the "ideal" position. The motion of Hilda is, of course, mainly determined by the gravitation of the Sun.

In the present paper, general computational algorithm for the planar restricted circular three-body problem in rotating synodic system is given in Section 2. The algorithm is applied for orbit determination of Hilda and Thule asteroids. The results are illustrated graphically in Section 3.

2. Equations of motion

We consider the planar restricted circular three-body problem in rotating synodic system (e.g., Szebehely, 1967) in which the two primaries are the Sun and Jupiter while the third infinitesimal third body is the asteroid. The equations of motion to be solved are

$$\dot{x} = u\,, \tag{1}$$

$$\dot{y} = v\,, \tag{2}$$

$$\dot{u} = -(1-\mu)\frac{x+\mu}{R_1^3} - \mu\frac{x-1+\mu}{R_2^3} + x + 2v\,, \tag{3}$$

$$\dot{v} = -(1-\mu)\frac{y}{R_1^3} - \mu\frac{y}{R_2^3} + y + 2u\,, \tag{4}$$

with

$$R_1 = \sqrt{(x-\mu)^2 + y^2}\,, \tag{5.1}$$

and

$$R_2 = \sqrt{(x+1-\mu)^2 + y^2}\,, \tag{5.2}$$

where dot denotes differentiation with respect to the time t, (x, y) are the coordinates of the third body, and μ denotes the mass of the smaller primary when the total mass of the primaries has been normalized to unity.

In these equations, the unit of length is the distance between the primaries, the unit of mass is the sum of the masses of the primaries. The unit of time is 1/n (n is the mean motion). Normally, n is expressed in a number of radians per second, hence in 1/sec. Its inverse 1/n is therefore expressed in seconds and may be interpreted as a unit of time.

3 .Orbit determination of Hilda and Thule asteroids

3.1 Initial conditions

Note that all the initial conditions of the following examples are taken from Hellings, (1994):

1. For the ideal orbit of Hilda, the stable non-librating triangle, one should start from $x_0 = -0.647717531$, $y_0 = 0.0$, $u_0 = 0.0$, $v_0 = -0.6828143998$.
   Using about 630 iterations with Dt = 0.02, a complete triangle taking three real revolutions of Hilda. The results are displayed in Figures (1).
2. For the ideal orbit of Thule, the initial conditions are
   $x_0 = -0.7997634829$, $y_0 = 0.0$, $u_0 = 0.0$, $v_0 = -0.3334548184$.
   The results are displayed in Figures (2).

3. If the position of the perihelion of Hilda is not exactly between the Sun and Jupiter, but makes an angle θ (say) with the line between the two primaries, the orbit is also stable. After three orbits of Hilda, the asteroid reaches its perihelion at a smaller angle θ than where it started (Hellings, 1994). The effect of Jupiter pulls the orbit in the direction of the ideal case. The resulting orbit is the typical triangle–like orbit, but now this triangle oscillates around the ideal position. In the case of Hilda, the libration, is about 40°. The period of one libration is known to be 270 years.

   For the case including the libration one has to start from the following

   values: $x_0 = -0.4952265404$, $y_0 = -0.4163448036$,

   $u_0 = 0.4389046359$, $v_0 = -0.5230661767$.

   Now a larger number of iterations is needed. When we take a time step of Dt = 0.05, about 2800 iterations are needed to observe one libration (see Figures 3). It was assumed in these calculations that the orbit of Jupiter is circular (in reality, e = 0.048) and that Hilda is moving in the orbital plane of the large planet (in reality there is an inclination of about 8°). Therefore, our results may deviate a little from the exact data. Finally, the value of μ is 0.000954786.

3.2 The results

It should be noted that, all the computations are performed using *Mathematica* 7. For clear illustrations of our analysis, the results are displayed graphically in the following manner:

1. Figures (1) for ideal orbit of Hilda without libration.
2. Figures (2) for ideal orbit of Thule without libration.
3. Figures (3) for orbit of Hilda with a libration of 40°.

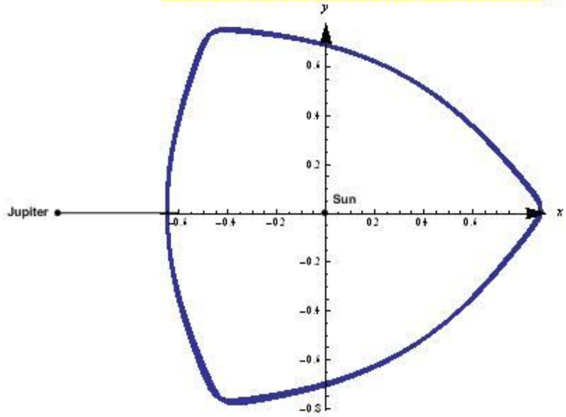


Fig.1 –1 : Ideal orbit of Hilda asteroid without libration

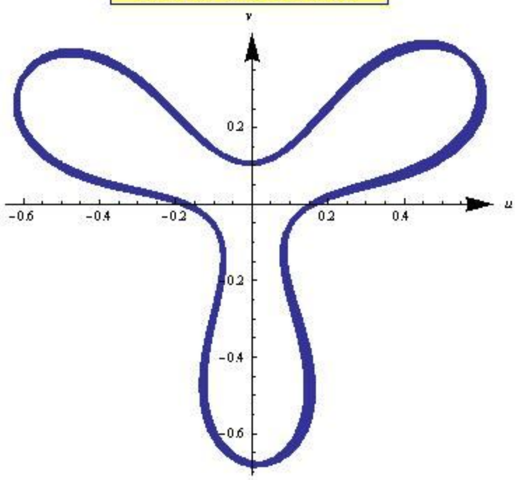
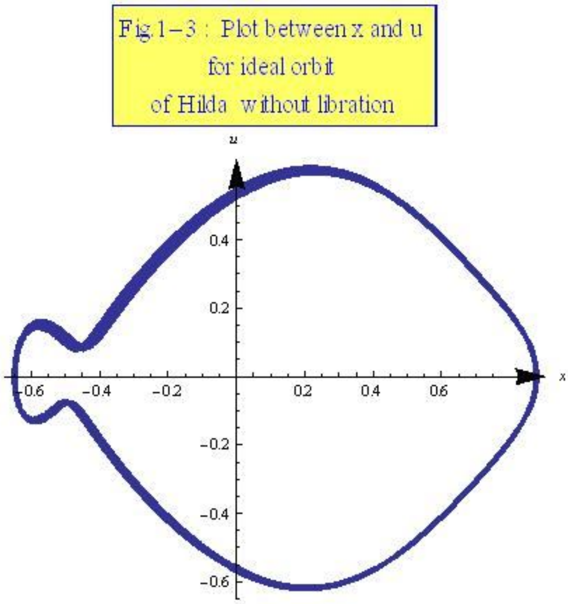


Fig. 1–2 :Plot between u and v for ideal orbit of Hilda without libration



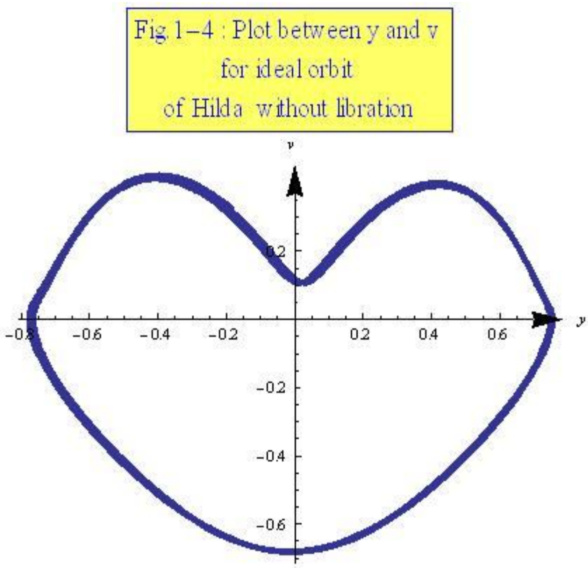


Fig.(1): Orbital analysis of ideal Hilda asteroid without libration.

Fig.2-1 : Ideal orbit of Thule asteroid without libration

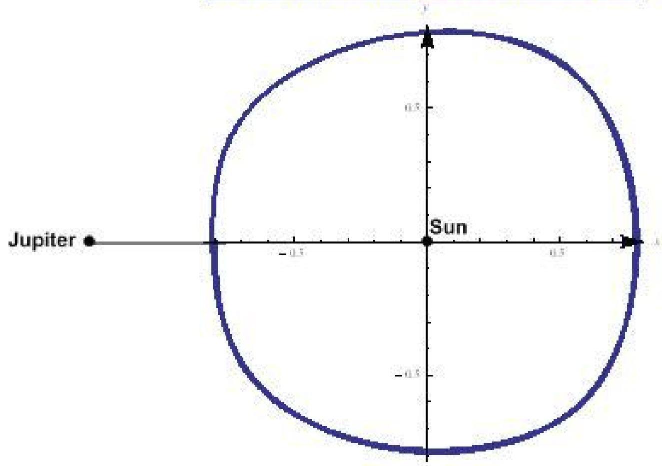
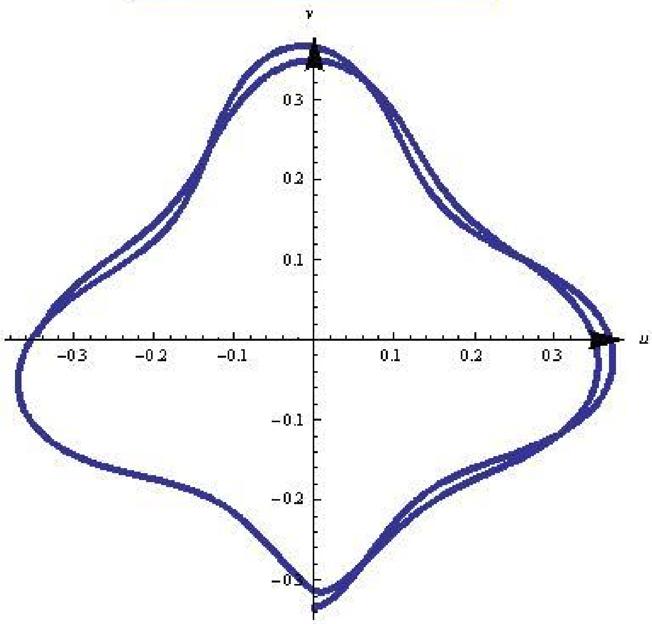


Fig.2-2 : Plot between u and v for ideal orbit of Thule without libration



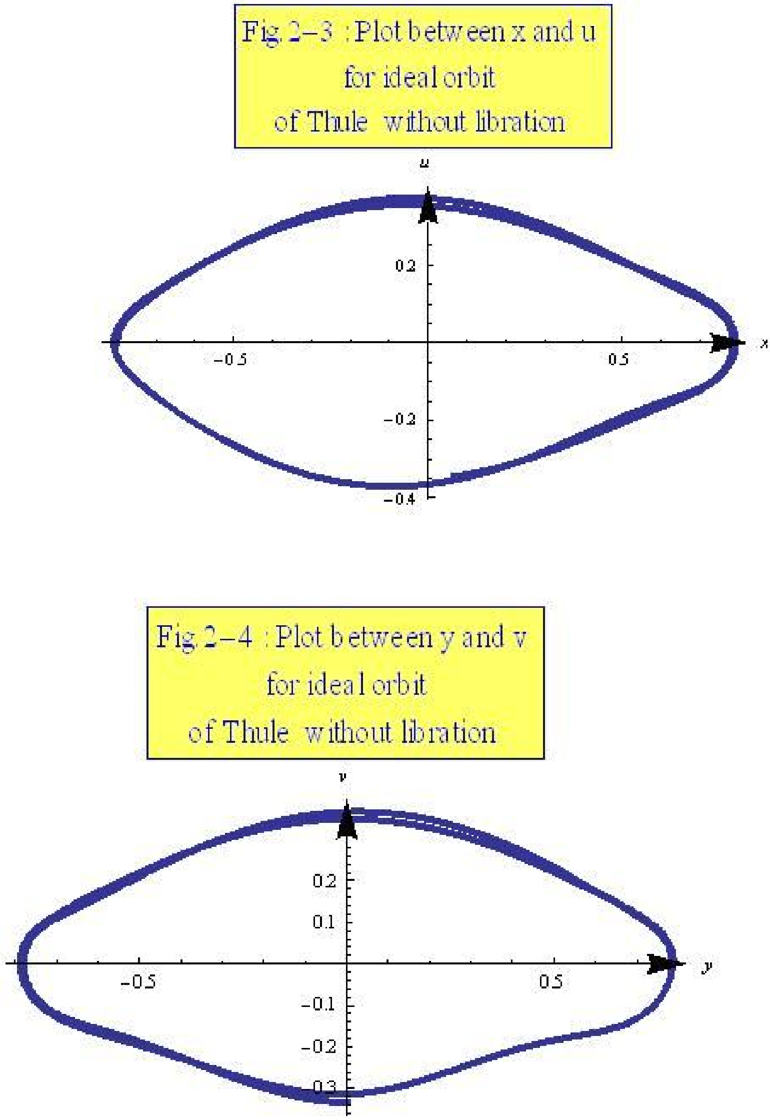


Fig.(2): Orbital analysis of ideal Thule asteroid without libration.

Fig.3− 1 : Orbit of Hilda asteroid,with libration 40°

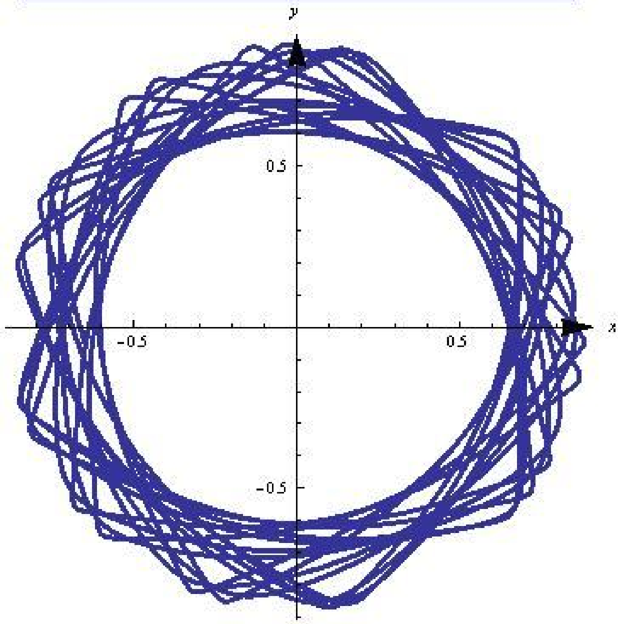
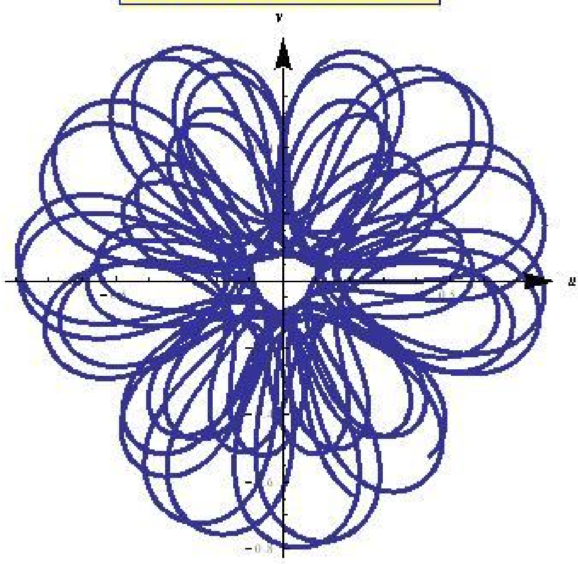
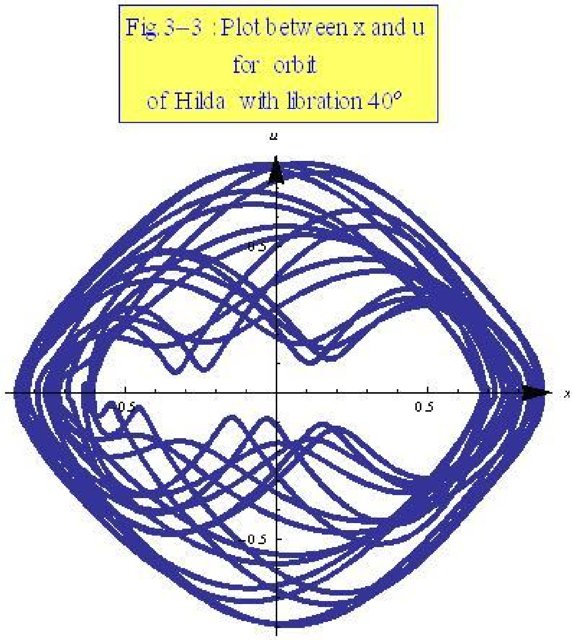


Fig.3−2 : Plot between u and v for orbit of Hilda with libration 40°





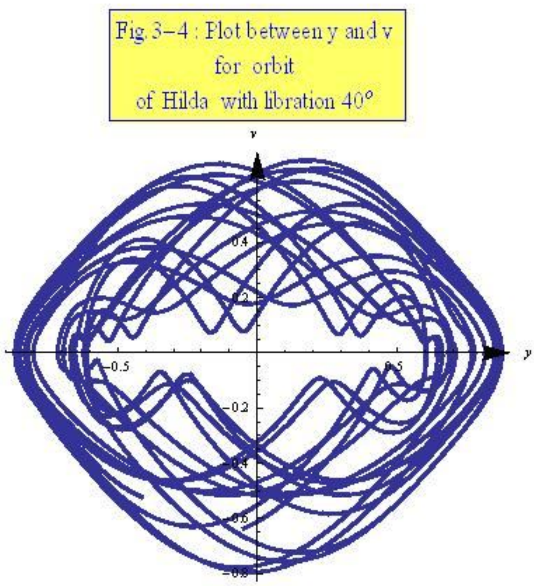


Fig.(3): Orbital analysis of Hilda asteroid with a libration 40°.

4. Conclusion

In the present paper, we have used the general computational algorithm for the planar restricted circular three-body problem in rotating synodic system. This algorithm is applied for determining the ideal orbit of Hilda and Thule asteroids. Then, the results obtained are illustrated graphically with libration of 40 degree and without.

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