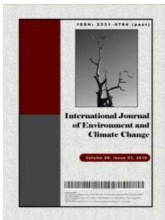

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Analytical River Routing with Alternative Methods to Estimate Seepage

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

Knowledge of flow exchange between surface and groundwater is of great importance for use of water resources. The determination of seepage between a stream and an underlying aquifer requires an accurate estimation of the river stage and of the head in the aquifer. An approach is presented to estimate analytically river flow and stage while using the SAFE conductance to calculate the seepage. A major contribution of this article lies in the methodology for river routing with its use of a modified Linear Reservoir model. The parameter C is related to discharge based on Manning's equation. That relation breathes into an empirical model a dynamic character. A second major contribution is to show that it is possible to simultaneously calculate river stage and aquifer head in the aquifer cell that contains the river. As a result iteration is not necessary to estimate that river cell head as river stage changes, as opposed to what is usually done in most numerical groundwater models. Iteration is still needed for the adjacent cells to the river cell. Because the influence of a change in the adjacent cell head on the river cell head is much delayed and attenuated the iteration is not sensitive to that change. A goal of this document is to show how that method can be used within a simple physically based routing procedure [1] to estimate the river stage that has a definite influence on seepage.

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Keywords: Stream-aquifer flow exchange; river routing; stream depletion; Leakance coefficient

1. INTRODUCTION

The practical contribution of this paper beyond what had been accomplished in previously published works needs some explanation. The author's interest has been to find the simplest possible way to improve the accuracy in MODFLOW (IWFM, MIKE-SHE, HydroGeoSphere, PIHM and other similar codes) to calculate the seepage from a river in saturated hydraulic connection with the aquifer without substantial changes in the codes.

The importance of the flow exchange between stream and aquifer is well recognized (e.g. [2,3]). The ability to predict accurately the flow exchange and the parameters and variables that determine it e.g. [4,5,6,7,8,9,10], is a necessary condition for successful planning, operations and abidance with law. That need has been recognized for a long time (e.g. [11]) by many scientists and it is becoming even more urgent. For example the recent California law, the Sustainable Groundwater Management Act, SGMA, September 16, 2014, requires that new agencies, Sustainable Groundwater Agencies, SGA, be created in all groundwater basins. These new agencies, which had to be created by June 30, 2017, must provide the Dept. of Water Resources by January 31, 2020 or 2022 a plan that would show that the SGA has taken action for sustainability of the groundwater resource. The majority of these plans will be supported by simulations with groundwater models.

For investigations over a very limited geographical extent it is possible to use very small-scale grids and thus obtain great accuracy for the results using numerical models. In large-scale regional studies using such small-scale grids is not practical. For example, in a California study for the Santa Rosa Plain Watershed Groundwater Management Plan ([12]) the square grid size is 660 feet and the area under investigation is 262 square miles. None of the rivers in the area have widths that exceed 100 feet. Thus all river reaches are included in cells that have dimensions far in excess of the river widths. In addition the water-table aquifer is treated as a single calculation layer even though its varying thickness always exceeds 90 feet. In such contexts the boundary condition to determine the seepage discharge is chosen to be of the third type (also named Cauchy, or General Head). The discharge is calculated as being proportional to the difference between the head in the river, h_S, and the head in the aquifer, h_f, at the center of the aquifer cell that includes the river (the river cell). The proportionality coefficient depends upon a "leakance coefficient", essentially treated in the past as an empirical parameter ([13]). Recently new procedures to estimate that coefficient that are based on physical principles were presented ([14,15,16,17]).

2. PREVIOUS WORK

In earlier articles ([14,15,17]), values of a dimensionless conductance were obtained analytically for a variety of cross-sections. This one-sided Stream-Aquifer Flow Exchange (SAFE) dimensionless conductance (in short the conductance), $\Gamma_{anis-stand-\Delta-rcl}$, accounts for degree of anisotropy, possible presence of a real clogging layer in the streambed, normalized wetted perimeter W_p^N, defined as wetted perimeter, W_P, divided by the aquifer thickness, $\bar{D}_{aq}$, $W_P^N = \dfrac{W_P}{\bar{D}_{aq}}$ (1) and maximum degree of penetration, d_p, defined as $H / \bar{D}_{aq}$ (2). H is the maximum depth of water in the river (the river stage), from the water surface to the streambed. K_H is the horizontal hydraulic conductivity. Δ stands for the excess far distance over the standard far distance, which in the present case (see Fig. 1) is: $\Delta = \dfrac{G}{4} - (2\dfrac{\bar{D}_{aq}}{\rho_{anis}} + B)$ (3) where G is the lateral grid size, B is the half width of the river and, $\rho_{anis} = \sqrt{\dfrac{K_V}{K_H}}$ (4) is a measure of anisotropy. The standard far distance is the minimum distance away from the river banks where the flow has essentially turned horizontal and the groundwater head behavior follows the Dupuit-Forchheimer approximation.

It is a goal of this document to show how that method can be incorporated within a simple physically based routing procedure for the river flow ([1]) to estimate accurately the river stage that has a definite influence on the seepage rate from the river. It is then possible to compare how various estimates of the leakance coefficient affect the estimates of the flow exchange.

Importantly because the procedure is analytical it is possible to jointly estimate the river flow and the seepage without the need for iteration between river stage and head in the river cell. That iteration is very sensitive and hence difficult because seepage is proportional to the difference between these two successively approximated heads. With the proposed procedure the iteration is between the river stage and the head in an adjacent cell. That adjacent cell being distant from the river the iteration is not as sensitive to that head.

3. GENERAL STATEMENT OF THE PROBLEM

To test the new procedure for the river flow and for the seepage a simple geometrical-configuration is used. Fig. 1 shows a cross-section of the system. The streambed could be a clogging layer, as shown in the figure below the river bottom. Given an inflow at the upstream end of a river reach water will propagate toward the downstream end, the wave getting delayed and attenuated as it travels the reach. On the way it may lose to or gain from the water-table aquifer.

For the description of the routing of flow in the river a modified version of the classic Linear Reservoir model ([18,19,20,21]) is used because the time constant is allowed to vary as a function of the discharge in the river. The relation gives to the originally empirical model a dynamic character because the time constant C is related to discharge based on Manning's equation (Morel-Seytoux, 2000), namely:

$$C = \left\{ \frac{3(n_M)^{3/5} W^{2/5} L}{5(S_R)^{3/10}} \right\} O^{-2/5} \tag{5}$$

This suggestion to incorporate dynamic characteristics within a conceptual routing model is not particularly new ([22,23]) though it has hardly been used. In Eq.(5) n_M is Manning's n, W is the river width, L is the river reach length, S_R is the river slope and O is the river outflow discharge. As that discharge increases, the time constant decreases. In other words the wave propagation celerity increases and any upstream disturbance, such as an increase in inflow, travels faster downstream. Eq. (5) provides a kinematic wave component for the flow while the incorporation of Eq.(5) in the LR. framework adds a diffusive component. This routing model

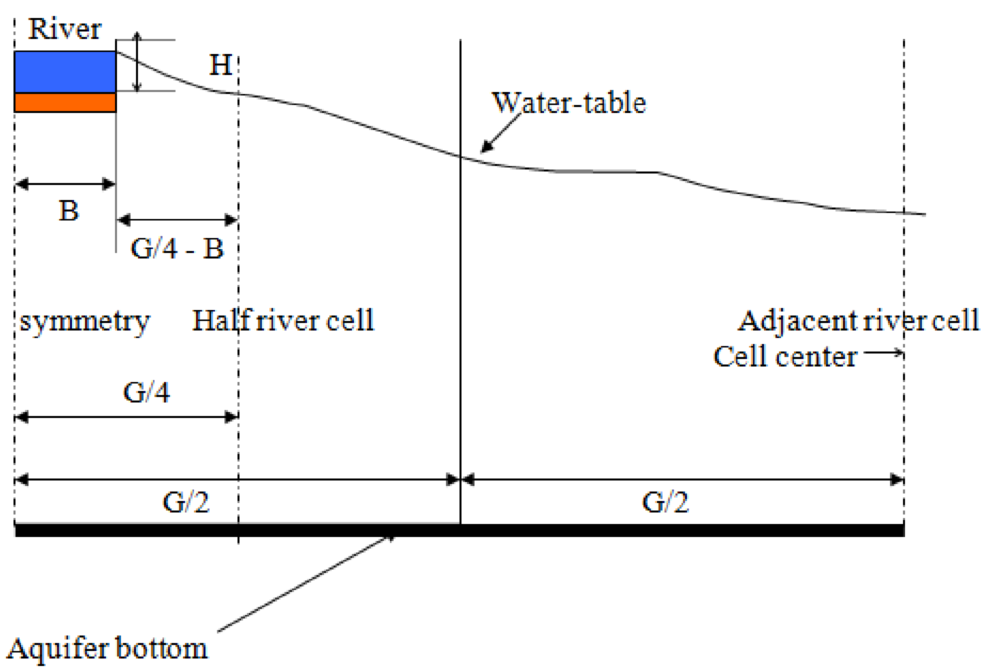

Fig. 1. Cross-section view of the river aquifer system showing the aquifer cells

for the river flow has been used for simulation of the Seine river basin upstream of Paris ([1,24, 25]).

4. MATHEMATICAL GENERAL FORMULATION

The mass balance governing equation for the classic Linear Reservoir (LR) routing model is:

$$\frac{dS}{dt} = \frac{d(CO)}{dt} = C\frac{dO}{dt} = I - O \qquad (6)$$

or $C\dfrac{dO}{dt} + O = I$ (7)

where $S = CO$ is storage, t is time, O is outflow rate (a discharge) and I is inflow rate. C is the LR "time constant, has dimension of time and in the classic LR model it is treated indeed as a constant in time. If the time constant is allowed to vary the governing equation takes the slightly different form:

$$C\frac{dO}{dt} + (1 + \frac{dC}{dt})O = I \qquad (8)$$

Now this equation does not include the seepage rate so that a more correct mass balance equation needs to be written as:

$$C\frac{dO}{dt} + (1 + \frac{dC}{dt})O = I - Q_S \qquad (9)$$

where Q_S is the seepage rate, algebraically counted positive if it is a gain for the aquifer, a loss from the river, thus a real seepage loss; if negative it is an aquifer return flow to the river. In short the word "seepage" is used in this document in lieu of the longer combination "flow exchange". It now remains to describe physically and mathematically the seepage rate.

5. DETERMINATION OF THE SEEPAGE RATE

Assuming a rectangular river cross-section and an elongated rectangular aquifer cell of smaller lateral side, G, and greater longitudinal side, L_R, one can relate the seepage to storage and river discharge. If the river cross-section is not rectangular an equivalent rectangular cross-section is one with the same maximum depth, which defines the head in the stream, and the same wetted perimeter, which conditions the area through which seepage takes place.

The total (i.e. two-sided) SAFE seepage discharge is:

$$Q_S = 2L_R K_H \Gamma(h_S - h_f) = T(h_S - h_f) \qquad (10)$$

with $K_L = K_H \Gamma$ and $T = 2L_R K_L$

where L_R is the river reach length (usually the same as the longitudinal size of the rectangular cell while G is the lateral size of the cell), W_p is the wetted perimeter of the cross-section, Γ written short for $\Gamma_{anis-stand-\Delta-rcl}$, as the case may be, is the one-sided SAFE dimensionless conductance ([14,15,16]), h_S is the head in the river and h_f is the head in the river cell. T has dimension of transmissivity (area per time). It plays the same role as C_{riv} in MODFLOW.

In the case of MODFLOW the total discharge is given by the expression:

$Q_S = L_R W_p \Lambda(h_S - h_f) = C_{riv}(h_S - h_f)$ (11) where Λ is the leakance coefficient

Comparing Eqs.(10) and (11) one can see that the SAFE leakance coefficient is simply:

$$\Lambda_{safe} = \frac{2K_H\Gamma}{W_p} = \frac{K_H\Gamma}{B+H} \qquad (12)$$

If heads are measured from the aquifer bottom then:

$$h_S = h_b + H \tag{13}$$

Where h_b is the elevation of the river bottom from a chosen datum (e.g. the aquifer bottom) and H is the river stage. Storage is $S = CO = WL_R H$ with $W = 2B$ being the river width. (A true or equivalent rectangular cross-section is assumed). In other words:

$$Q_S = T(h_b + H - h_f) = \frac{K_L C}{B} O - T\Delta h_f \quad (14) \quad \text{with } \Delta h_f = h_f - h_b \tag{15}$$

Any head measured from the river bottom is preceded by the symbol Δ. Substituting in Eq.(9) yields:

$$C\frac{dO}{dt} + [1 + \frac{dC}{dt} + \frac{K_L C}{B}]O = I + T\Delta h_f \tag{16}$$

For simplicity in writing we define the average value of $\frac{dC}{dt}$ over the period of time (time step) $\frac{dC}{dt} = \lambda$ (17a) and similarly $\frac{K_L \bar{C}}{B} = \mu$ (17b). Note that μ may change because both K_L and C will change.

With these notations the governing equation becomes:

$$(C_i + \lambda t)\frac{dO}{dt} + [1 + \lambda + \mu]O = I(t - \tau) + T\Delta h_f \tag{18a}$$

or setting $\delta = 1 + \lambda + \mu$

$$(C_i + \lambda t)\frac{dO}{dt} + \delta O = I(t - \tau) + T\Delta h_f \tag{18b}$$

or

$$\frac{(C_i + \lambda t)}{\delta}\frac{dO}{dt} + O = \frac{1}{\delta}\{I(t - \tau) + T\Delta h_f\} \tag{18c}$$

with C_i the initial value of C and where we have added the possibility that an inflow change at the upstream end of the reach may not be felt downstream without a delay τ. This is adding a hyperbolic aspect to the basic routing model that otherwise would be purely parabolic. In Eq.(18) appears the head in the river cell, Δh_f.

The solution (see Appendix 1 in online supporting information) is:

$$O(n) = \rho_{On} O(n-1) + [1 - \sigma_{On}]\frac{\bar{I}(n)}{\delta_n} + [\sigma_{On} - \rho_{On}]\frac{\bar{I}(n-1)}{\delta_n} + \beta_{On}\frac{T_n}{\delta_n}\Delta h_f(n) + \alpha_{On}\frac{T_n}{\delta_n}\Delta h_f(n-1) \tag{19}$$

with $\rho_{On} = (\frac{C_n}{C_{n-1}})^{p_n}$ (20a)

$$C_{n\tau} = C_i + \lambda\tau \quad (20b)$$

$$\sigma_{On} = (\frac{C_n}{C_{n\tau}})^{p_m} \quad (20c)$$

$$\alpha_{On} = [(1-\rho_{On})(1+\frac{C_{n-1}}{\lambda_n+\delta_n}) - \frac{\delta_n}{\lambda_n+\delta_n}] \quad (20d)$$

$$\beta_{On} = \frac{1}{(\lambda_n+\delta_n)}[\delta_n - (1-\rho_{On})C_{n-1}] \quad (20e)$$

However that head is not likely to be a decision variable but rather a state variable. If that is the case the real decision variable is the head in the adjacent cell, Δh_{adj}. Thus it is necessary to find how the river cell aquifer head responds to the head in the adjacent cell.

6. DETERMINATION OF THE AQUIFER HEAD IN THE RIVER CELL

The head associated with the center of the river cell, as calculated by the finite difference method, is not the punctual head at that location, but the average head over the entire river cell. Thus the head associated with the center of each half-cell on either side of the center of symmetry is the same as the head in the (full) river cell. To guarantee that the flow between the center of the half river cell and the adjacent cell is horizontal one must require (see Fig. 1) that $\frac{G}{4} - B \geq 2\frac{D_{aq}}{\rho_{anis}}$ (21a) or $G \geq 8\frac{D_{aq}}{\rho_{amis}} + 4B$ (21b). The excess distance over the minimum far distance necessary to assure horizontal flow is:

$$\Delta = \frac{G}{4} - (2\frac{D_{aq}}{\rho_{anis}} + B) \quad (21c)$$

Mass balance for the aquifer in either half river cell (right or left), is as expected:

$$\phi_e \frac{G}{2} L_R \frac{\partial \Delta h_f}{\partial t} = K_L L_R(\Delta h_S - \Delta h_f) - K_H \bar{D}_{aq} L_R \frac{(\Delta h_f - \Delta h_{adj})}{G/4 + G/2} \quad (22a)$$

In other words storage in the (aquifer) half river cell increases if the river seepage (first term on the right) exceeds the lateral flow to the adjacent cell. Eq.(22a) is a differential equation, not a finite difference equation. When solved analytically it provides the spatial average value of the head in the half river cell continuously in time. In addition the seepage rate is not estimated by a finite difference vertical approximation of the flux across the bottom of the river but by an estimate of seepage that accounts for the turning, convergence and divergence of the flow between the river wetted perimeter and the center of the half river cell

([14];[15];[17]). Because the location of the center of the half river cell is far enough away from the river bank the flow beyond that point is horizontal and the flux across the boundary with the adjacent cell is expressed by Darcy's law for horizontal flow. On the other hand the head in the adjacent cell would be expected to be obtained by solution of the system of finite difference equations for the overall configuration of the regional aquifer. The method of solution combines a local analytical technique in the vicinity of the river with a regional finite difference formulation for the overall aquifer system. In that way a local finite difference discretization near the river is avoided while still representing the full local 2-dimensional aspect in the solution. Unnecessary numerical work is avoided and accuracy is improved.

G is the lateral size of the rectangular cell, ϕ_e is the specific yield (effective porosity) of the river cell aquifer, $\bar{D}_{aq}$ is the average aquifer thickness in the vicinity of the river and Δh_{adj} is the head in the adjacent cell on the right or left side. Bringing all the heads, Δh_f, on the left hand side yields:

$$G\phi_e \frac{\partial \Delta h_f}{\partial t} + 2[K_L + \frac{4}{3}\frac{D_{aq}}{G}K_H]\Delta h_f = 2K_L \Delta h_S + \frac{8}{3}\frac{D_{aq}}{G}K_H \Delta h_{adj} \qquad (22b)$$

$$\text{or } G\phi_e \frac{\partial \Delta h_f}{\partial t} + 2[K_L + \frac{4}{3}\frac{D_{aq}}{G}K_H]\Delta h_f = K_L \frac{CO}{BL_R} + \frac{8}{3}\frac{D_{aq}}{G}K_H \Delta h_{adj} \qquad (22c)$$

and with: $C_f = \dfrac{G\phi_e}{2[K_L + \frac{4}{3}K_H \bar{D}_{aq}/G]}$ (23a) dimension of time, $\rho_f = e^{-\frac{1}{C_f}}$ (23b)

$$C_S = C_{stage} = \frac{K_L}{K_L + \frac{4}{3}K_H \bar{D}_{aq}/G} \quad (23c) \qquad C_{adj} = \frac{\frac{4}{3}K_H \bar{D}_{aq}/G}{K_L + \frac{4}{3}K_H \bar{D}_{aq}/G} \qquad (23d)$$

both dimensionless, one obtains:

$$C_f \frac{\partial \Delta h_f}{\partial t} + \Delta h_f = \gamma_O O + C_{adj}\Delta h_{adj} \quad (24) \qquad \text{with } \gamma_O = C_S \frac{C}{BL_R} \qquad (25)$$

dimension inverse of transmissivity. This equation has the same structure as the classic LR.

The solution for this equation (Appendix 1 in online supplementary information) is:

with $\alpha_f = [C_f(1- \ \rho_f) - \rho_f]$ (26a) and $\beta_f = [1 - C_f([1-\rho_f)]$ (26b)

$$\begin{aligned}\Delta h_f(n) &= \rho_f \Delta h_f(n-1) + \beta_f \gamma_{On} O(n) + \alpha_f \gamma_{On-1} O(n-1) \\ &+ \beta_f C_{adj}\Delta h_{adj}(n) + \alpha_f C_{adj}\Delta h_{adj}(n-1)\end{aligned} \qquad (27)$$

7. FINAL EQUATION FOR THE OUTLFLOW AFTER ELIMINATION OF THE RIVER CELL HEAD

The governing equation (see Appendix 1 in online supporting information for derivations) at end of period (day) n for the river flow is:

$$[1-\beta_{On}\frac{T_n}{\delta_n}\beta_f\gamma_{On}]O(n)=[\rho_{On}+\beta_{On}\frac{T_n}{\delta_n}\alpha_f\gamma_{On-1}]O(n-1)$$
$$+[1-\sigma_{On}]\frac{\overline{I}(n)}{\delta_n}+[\sigma_{On}-\rho_{On}]\frac{\overline{I}(n-1)}{\delta_n}$$
$$+[\beta_{On}\frac{T_n}{\delta_n}\rho_f]\Delta h_f(n-1)$$
$$+[\beta_{On}\frac{T_n}{\delta_n}\alpha_f C_{adj}]\Delta h_{adj}(n-1)$$
$$+[\beta_{On}\frac{T_n}{\delta_n}\beta_f C_{adj}]\Delta h_{adj}(n) \quad (28)$$

with $C_n = C_{n-1} + \lambda_n$ (29a) $\delta_n = 1+\lambda_n+\mu_n$ (29b) $p_n = -\frac{\delta_n}{\lambda_n}$ (29c)

$$\rho_{On}=(\frac{C_n}{C_{n-1}})^{p_n} \quad (25d)$$

$C_{\tau n} = C_{n-1}+\lambda_n\tau$ (29e) $\sigma_{On} = (\frac{C_n}{C_{\tau n}})^{p_n}$ (25f) $\rho_f = e^{-\frac{1}{C_f}}$ (29g)

$\alpha_f = [C_f(1-\ \rho_f)-\rho_f]$ (29h) $\beta_f = [1-C_f([1-\rho_f)]$ (29i)

Note that the influences of the decision variables, namely the inflows for the current period $\overline{I}(n)$ and for the previous period, $\overline{I}(n-1)$ and the heads in the adjacent cells, $\Delta h_{adj}(n)$ and $\Delta h_{adj}(n-1)$, are explicitly described. Similarly the influence of the initial conditions at the beginning of the period is shown in the terms including $O(n-1)$ and $\Delta h_f(n-1)$.

(In statistical parlance this is a typical Autoregresive Moving Average model ([26]; multiple variables ARMA(1,2), except that in the statistical literature the coefficients are not allowed to vary in time).

8. PURPOSE AND TYPES OF RUNS

The purpose of the runs is to compare several estimations of the leakance coefficient and to assess the resulting differences on the values of seepage. It is also to demonstrate that the head in the river cell is primarily dependent upon the seepage amount and not so strongly influenced by the head in the adjacent cell. Table 1 shows the different types of scenario using different ways to estimate the leakance coefficient

Type (scenario) number Leakance coefficient

1. Use of the SAFE dimensionless conductance $\Gamma_{anis-stand-\Delta-rcl}$ and as a result a variable leakance coefficient
2. An average value of the above leakance coefficient over the period of simulation
3. Zero. The river bottom is impervious
4. The maximum value of the leakance coefficient in scenario 1
5. The minimum value of the leakance coefficient in scenario 1

To ease interpretation of the results in the figures the same color is associated with a run type. Type 1 is black, type 2 is blue, type 3 is green, type 4 is red, and type 5 is magenta.

9. THE RIVER AQUIFER SYSTEM USED IN THIS STUDY

The river and aquifer system used in this document is not a specific real system though parameters for the river correspond to a real river. Thus it may be viewed as a "theoretical" system. In studies to be performed for SGMA a real groundwater basin will be simulated using a groundwater code, often MODFLOW or IWFM. Following calibration, when making future predictions under different management strategies, the model used will not be the real system but a calibrated approximation of the real system, in other words itself a "theoretical" model. Similarly the excitations will not be real observed ones but conceptual ones, such as releases from an upper dam and pumping from wells that did not exist during the historical calibration period, etc. Thus the inflow pattern used in the present study can be thought to be the controlled release from a dam upstream from the chosen Marne river reach. The excitation pattern in the adjacent cell can be thought to be the result of a pattern of pumping and/or recharge in the system. For easier interpretation of the results that pattern is chosen to be simple. Similarly the release from the upper reservoir is chosen to represent a situation of a flood followed by a long recession.

10. NUMERICAL EXAMPLE

The parameters for the system are indicated in Table 1.

If not specified all lengths are in meters and the period of time is the day. Parameters for the river correspond to an upper reach of the Marne river, a tributary to the Seine river in France ([24]; [25]).

Table 1. Parameters for river geometry, river dynamics, aquifer characteristics and initial conditions

Parameters	L(km)	B	G	K_H	D_{brb}	ϕ_e	anisotropy	τ
Case 1	40.00	10.0	350.0	20.0	10.0	0.20	0.10	0.00
Parameters	H_{ini}	Δh_f^{ini}	Δh_{adj}^{ini}	$H_{\max}$	O_{ini} (cms)	C_{ini}		
	1.4610	1.4610	1.4610	3.00	78.0	0.1734		
Parameters	K_{rcl}	e_{rcl}	h_{ce}	Slope	Manning's n			
	0.10	0.00	0.30	0.00087	0.03333			

Table 2. Evolution of inflow discharge (cms) with time (day)

Period	$n \le 16$	$17 \le n \le 70$
$\bar{I}(n)$	$320 - 240\cos[\lvert(n-1)\pi/18)\rvert]$	$\bar{I}(16)e^{-\frac{(n-16)}{20}}$

Table 3. Evolution of head in the adjacent cell with time (days)

Period	$n \le 2$	$n = 3$	$4 \le n \le 20$	$21 \le n \le 70$
$\Delta h_{adj}(n)$	1.4610	0.0	-2.0	2.0

11. NUMERICAL EXAMPLE. CASE OF ANISOTROPY AND NO CLOGGING LAYER

Because the magnitude of seepage is small compared to the river discharge, figures showing the outflows in cases there is seepage or no seepage will not provide much information. In Fig. 2 only the outflows for runs of types 1 and 3 (no seepage) are shown. The outflows for the other 3 types were visually the same. One can hardly discern a difference between the outflows. In practice what this means is that when calibrated flows show a certain amount of difference compared to observations, it is almost impossible to know if the difference is partly due to error in estimation of seepage, due to measurement errors or due to the calibration.

Fig. 3 shows the seepage rates for the different types. The discharges in ordinate axis in the figures are in cubic meters per second (cms).

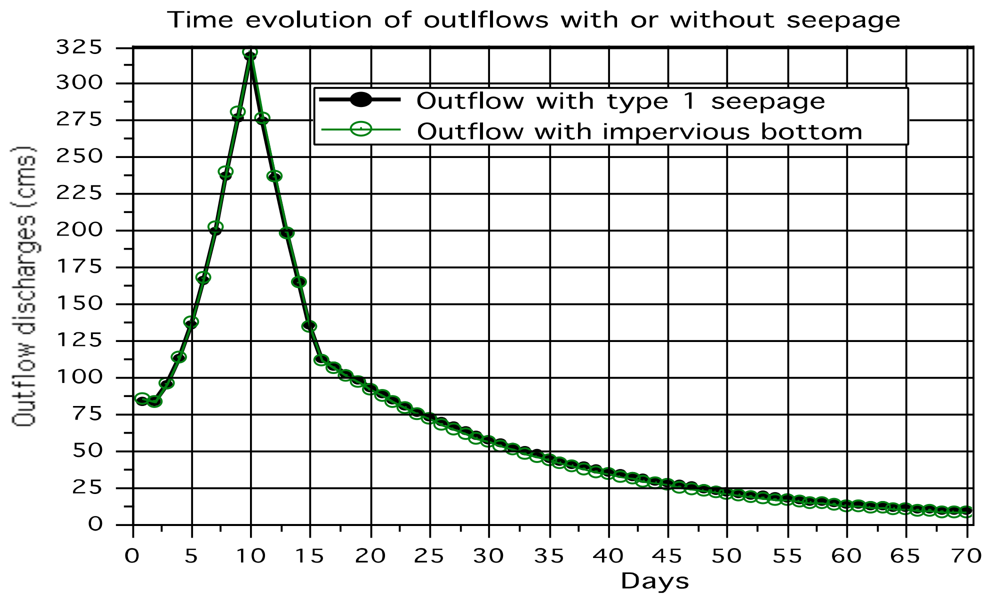

Fig. 2. Outflow pattern between case of seepage or no seepage

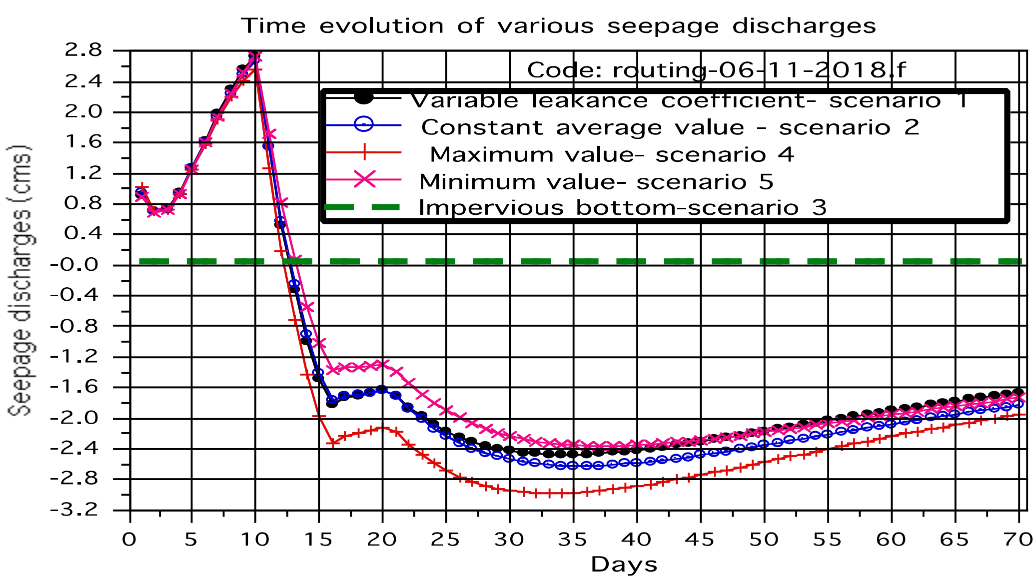

Fig. 3. Seepage patterns for the various types of runs

Type 2 does not match perfectly type 1, the benchmark with the correct analytical answer. The difference is quite small but can be of the order 0.1 cubic meters per second (cms) or 3.6 cubic feet per second or 0.14 cfs/mile in this case. In the Poudre river area of Colorado a rough estimate for leakage from unlined irrigation canals is 1 cfs/mile. With type 4 the differences are larger and can be as much as 0.5 cms or 18 cfs or 0.75 cfs/mile. It is now of the (gross) order of magnitude for seepage for unlined irrigation canals in Colorado.

As shown in Fig. 4 the leakance coefficients can vary significantly with time and with the scenario.

An average value for the leakance does not work perfectly well even if calibrated exactly on the average value of the correct leakance variation with time. Theimplication is that using a constant leakance coefficient for a river reach, as typically obtained by calibration, may not provide an accurate estimate of the seepage unless close to the mean value for type 1. This is the case for type 4 where the leakance coefficient is constant but clearly too large leading to differences in seepage, actually excessive return flows, between days 15 to 70. During those 55 days the ratio of the return flow between type 4 and type 1 is 1.1148 at day 35 and 1.1235 at day 70. These two values are close to the ratio of the leakance coefficients at those times respectively, 1.1075 and 1.1970, but not identical. The seepage values depend upon the leakance coefficients but also upon the head differences between the river stage and the heads in the river cell, both of which are affected by the river flow and by the head in the adjacent aquifer cell. There is a strong compensation effect because if under one scenario there is more actual seepage than for another scenario the head in the river cell will rise more in that scenario than in the other scenario thus reducing the difference in head for the next day and in seepage. This is particularly true if the river cell has difficulty to move water toward the adjacent cell, which is the situation if the head in the adjacent cell exceeds that in the river cell.

Finally Fig. 5 provides the values of river stage and heads in the aquifer cells.

In case of no seepage it is clear that the head in the adjacent cell will tend to influence more strongly the head in the river cell but still slowly, if for no other reason than it is far away. If the head in the adjacent aquifer cell remains the same eventually the head in the river cell will take its value. On the other hand when there is seepage as with types 1, 2, 4 and 5, very clearly the head in the river cell is far more influenced by the seepage, that is by the river stage, than it is by the head in the distant adjacent cell. That suggests the merit of a joint analytical evaluation of river stage and river cell head and iteration only for the head in the adjacent cell.

Full results for this numerical example are provided in tabular form in online Appendix 2.

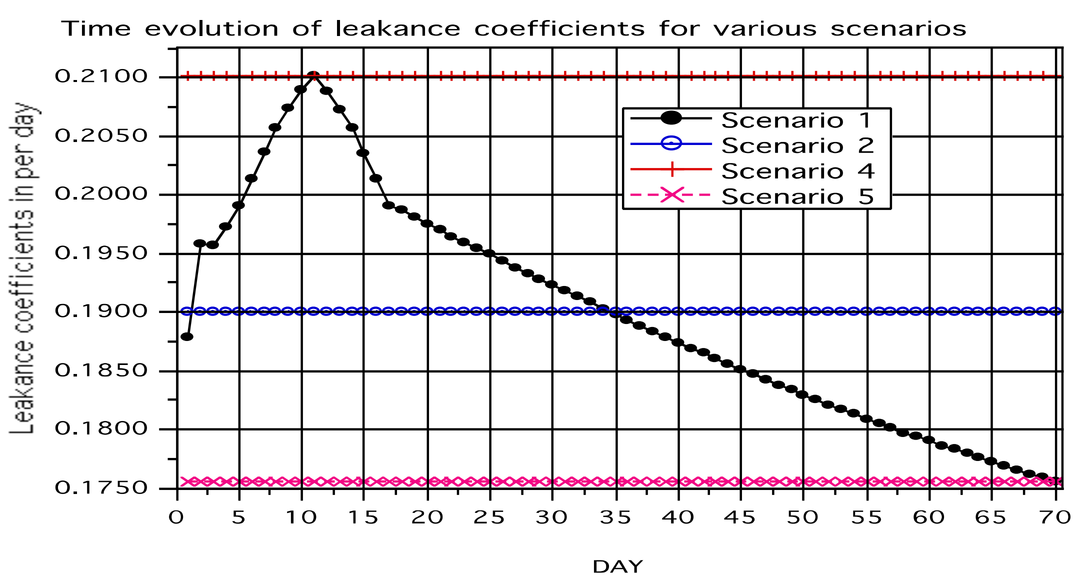

Fig. 4. Leakance coefficients used for the various run types

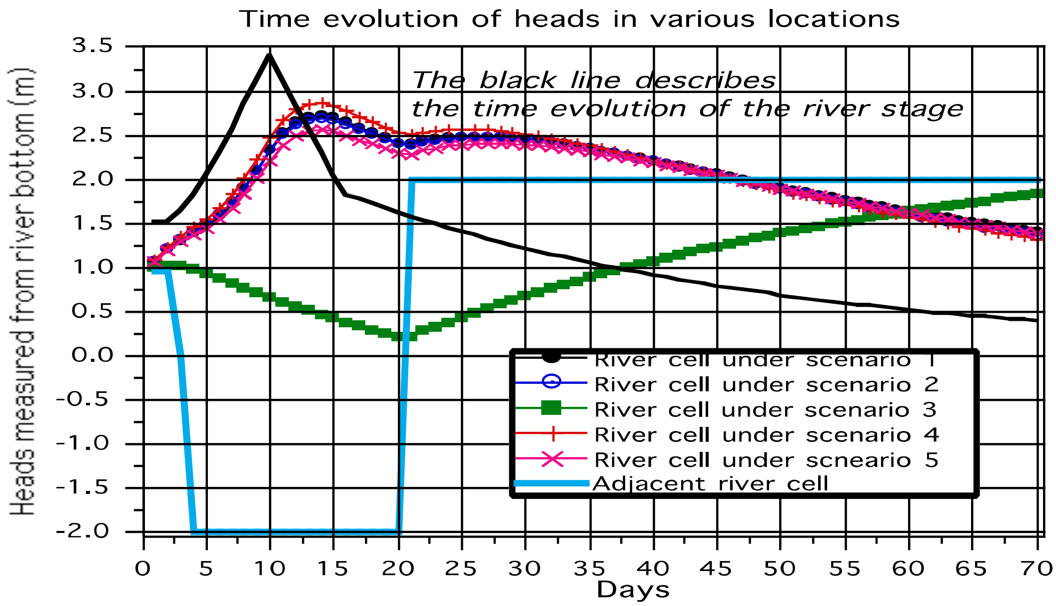

Fig. 5. River stage and aquifer head patterns for the 5 types of runs

12. DISCUSSION

There is a complex relation between seepage, river stage, heads in river cell and in the adjacent cells. That relation is affected by the grid size. Ideally the correct amount of flow exchange between the river and the water table aquifer should not be affected by the grid size selected in the finite difference approximation of the system [27]. In the numerical example shown in this article the grid size is still relatively small but getting close to the typical size used in large-scale regional studies. As a result the influence of the adjacent aquifer cell head is felt relatively rapidly though not too strongly. With a larger grid size that influence would be felt even more slowly and less strongly. In other words river cell head influences the seepage far more rapidly and strongly than the adjacent cell head. The process of iteration for river stage and river cell head to determine seepage, as used in typical finite difference or element approaches, can have great difficulty to converge. If on the other hand the relation between river and river cell is done analytically, as proposed, the process of iteration would be between river stage and adjacent aquifer cell head and its convergence will be more rapid.

For ease of introduction to new concepts the present article describes a system with symmetry of heads in the adjacent cells. However that limitation can be removed and the resulting equations are not more complicated, as previously indicated [16].

In most groundwater models used in regional studies the criterion for incipient desaturation of the stream-aquifer hydraulic connection is fairly crude and the description of the seepage under transient unsaturated connection is quite empirical. With the analytical approach a physical description of seepage under unsaturated connection can be secured without relying on a costly numerical solution of an unsaturated flow equation (e.g. Richards' equation).

13. CONCLUSION

The leakance coefficient cannot be considered constant in time as the excitation patterns of river flow and adjacent cell head change with time. Even if a constant value, obtained as the precise average value of the exact leakance coefficient variation in time, is used, the estimation of seepage will not be perfectly accurate as shown for example in Fig. 3. Naturally it is not likely that a calibrated value of a constant leakance coefficient will happen to be the precise value of the average of the correct variation of the exact leakance coefficient with time, and the errors may be as much as shown with type 4 or 5 in Figs. 4 and 7, possibly less or more depending on circumstances.

Using an analytically derived routing procedure has the advantage that it allows to obtain an explicit relation between river outflow, a state variable, and the excitation of river inflow and adjacent aquifer head, the decision variables. The disadvantage of the classical Linear Reservoir approach is that it assumes a time invariant behavior of the system. With the proposed approach the system is represented as a time-variant linear system. The advantage is that the system remains linear but is able to account indirectly for the nonlinearity existing between river flow and river stage [1].

The explicit character of the relation between outflow and the excitation variables would make it easy, or at least somewhat easier, to determine the probability distribution (or more humbly the variance) of outflow in terms of the distribution of the external excitations or of the various parameters which appear in the coefficients of the relation. With greater awareness and concerns with climate change and uncertain population growth, decision makers would like to be informed of the probable consequences of the considered management decisions for the future.

Maybe more importantly if such analytical scheme was used in a numerical model the iterative procedure to determine in turn river stage and river cell head would be replaced by the determination in turn of river stage and adjacent cell head. That procedure, as suggested by the simulations in this article, should converge more rapidly.

Naturally the behavior of a surface-ground waters system subject to diverse river flows and aquifer withdrawal or recharge rates will vary immensely from one region to another, from a season to another. What this article suggests is that the continuing practice of using a constant empirical leakance coefficient, even when calibrated, will lead to errors in the evaluation of the seepage rates, the magnitude of which could be small or large. It would be wise to modify that practice and an alternative is presented in this article.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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APPENDIX 1. To routing paper

Solution to the Governing Equation for Outflow:

$$(C_i + \lambda t)\frac{dO}{dt} + [1 + \lambda + \mu]O = I(t - \tau) + T\Delta h_f \quad \text{(a)}$$

For simplicity in writing let $\delta = [1 + \lambda + \mu]$ (b)

Eq.(a) becomes: $(C_i + \lambda t)\dfrac{dO}{dt} + \delta O = I(t - \tau) + T\Delta h_f$ (c)

There are two different types of excitation. One corresponds to a time delay in an average value of the excitation and one to a time linear variable one.

So let us find the solutions separately to two types of equations, namely first:

$(C_i + \lambda t)\dfrac{dU}{dt} + \delta U = E(t - \tau)$ (A1) and second

$$\frac{(C_i + \lambda t)}{\delta}\frac{dU}{dt} + U = E^o + (E^\upsilon - E^o)t \quad \text{(B1)}$$

Solution to Eq.(A1)

The solution is of the form, in the interval of time $0 \le t \le \tau$:

$U(t) = A + D[\dfrac{C_i + \lambda t}{C_i}]^p$ (A2) and thus $\dfrac{dU}{dt} = pD[\dfrac{C_i + \lambda t}{C_i}]^{p-1}\dfrac{\lambda}{C_i}$ (A3)

Substitution in Eq.(A1) yields:

$$pD\lambda[\frac{C_i + \lambda t}{C_i}]^p + \delta[A + D[\frac{C_i + \lambda t}{C_i}]^p] = \bar{E}(0) \quad \text{(A4)} \quad 0 \le t \le \tau$$

where $\bar{E}(0)$ represents the average excitation for the previous period. For the left hand side to match the right hand side the coefficient of the function of time must be zero. Thus:

$pD\lambda + \delta D = 0$ which yields $p = -\dfrac{\delta}{\lambda}$ (A5)

(note that since $p + 1 = -\dfrac{(1 + \mu]}{\lambda}$ it is always of the opposite sign of λ)

and $(p+1)\lambda = -(1+\mu)$ is always negative. Similarly $A = \dfrac{\bar{E}(0)}{\delta}$ (A6).

Using these values in Eq.(A2) yields:

$$U(t) = \frac{E(0)}{\delta} + D[\frac{C_i + \lambda t}{C_i}]^p \qquad \text{(A7)}$$

At time zero the value of U is the initial value U_i. Thus $D = U_i - \dfrac{E(0)}{\delta}$

Finally the expression for U(t) is:

$$U(t) = U_i[\frac{C_i + \lambda t}{C_i}]^p + \{1 - [\frac{C_i + \lambda t}{C_i}]^p\} \frac{\bar{E}(0)}{\delta} \quad \text{(A8a)} \text{ for } 0 \le t \le \tau$$

In particular at time $t = \tau$ then we have :

$$U_\tau = U_i[\frac{C_i + \lambda\tau}{C_i}]^p + \{1 - [\frac{C_i + \lambda\tau}{C_i}]^p\} \frac{\bar{E}(0)}{\delta}$$

or setting $C_i + \lambda\tau = C_\tau$: $\quad U_\tau = U_i(\dfrac{C_\tau}{C_i})^p + [1 - (\dfrac{C_\tau}{C_i})^p]\dfrac{\bar{E}(0)}{\delta}$ (A9)

For period n, defining : $\rho_{\tau n} = (\dfrac{C_{\tau n}}{C_{n-1}})^{p_n}$ (A10) and $\delta_n = 1 + \lambda_n + \mu_n$ (A11)

$$U_{\tau n} = \rho_{\tau n} U(n-1) + (1 - \rho_{\tau n}) \frac{\bar{E}(n-1)}{\delta_n} \quad \text{(A12)}$$

For the remainder of the period, during the time interval $\tau \le t \le 1$, we have:

$$U(t) = U_\tau[\frac{C_i + \lambda t}{C_\tau}]^p + \{1 - [\frac{C_i + \lambda t}{C_\tau}]^p\} \frac{\bar{E}(1)}{\delta} \quad \text{(A13)}$$

For period n, with $\sigma_n = (\dfrac{C_n}{C_{\tau n}})^{p_n}$ (A14)

$$U(n) = \sigma_n U_{\tau n} + (1 - \sigma_{\tau n}) \frac{\bar{E}(n)}{\delta_n} \quad \text{(A15)}$$

Elimination of U_τ yields:

$$U(t) = \{U_i(\frac{C_\tau}{C_i})^p + [1-(\frac{C_\tau}{C_i})^p]\frac{\bar{E}(0)}{\delta}\}[\frac{C_i+\lambda t}{C_\tau}]^p + \{1-[\frac{C_i+\lambda t}{C_\tau}]^p\}\frac{\bar{E}(1)}{\delta} \quad \text{(A16)}$$

In particular at time period n, with $t=1$:

$$U(n) = (\frac{C_n}{C_{n-1}})^{p_n}U(n-1) + [1-(\frac{C_n}{C_{\tau n}})^{p_n}]\frac{\bar{E}(n)}{\delta_n} + [(\frac{C_n}{C_{\tau n}})^{p_n} - (\frac{C_n}{C_{n-1}})^{p_n}]\frac{\bar{E}(n-1)}{\delta_n} \quad \text{(A17)}$$

Defining, as before, $\rho_{Un} = (\frac{C_n}{C_{n-1}})^{p_n}$ (A18) $\sigma_{Un} = (\frac{C_n}{C_{\tau n}})^{p_n}$ (A19) then we obtain: $U(n) = \rho_{Un}U(n-1) + [1-\sigma_{Un}]\frac{\bar{E}(n)}{\delta_n} + [\sigma_{Un} - \rho_{Un}]\frac{\bar{E}(n-1)}{\delta_n}$ (A20)

Solution to Equation (B1)

The equation is of the type: $\frac{(C_i+\lambda t)}{\delta}\frac{dU}{dt} + U = E^o + (E^\upsilon - E^o)t$ (B1)

The time "constant" varies linearly with time and so does the excitation.

Solution is of the form: $U(t) = A + Mt + D(\frac{C_i+\lambda t}{C_i})^p$ (B2)

The derivative is: $\frac{dU}{dt} = M + D\frac{p\lambda}{C_i}(\frac{C_i+\lambda t}{C_i})^{p-1}$ (B3)

Substitution in Eq.(B1) yields:

$$\frac{(C_i+\lambda t)}{\delta}\{M + D\frac{p\lambda}{C_i}(\frac{C_i+\lambda t}{C_i})^{p-1}\} + \{A + Mt + D(\frac{C_i+\lambda t}{C_i})^p\} = E_o + (E_\nu - E_o)t \quad \text{(B4)}$$

For the solution to be satisfied requires that:

$\frac{C_i M}{\delta} + A = E_o$ (B5) $(\frac{\lambda}{\delta}+1)M = (E_\nu - E_o)$ (B6) $D[\frac{p\lambda}{\delta}+1] = 0$ (B7)

$M = \delta\frac{(E_\nu - E_o)}{(\lambda+\delta)}$ (B8) $A = E_o - \frac{C_i M}{\delta} = E_o - C_i\frac{(E_\nu - E_o)}{(\lambda+\delta)}$ (B9) $p = -\frac{\delta}{\lambda}$ (B10)

Substitution of these parameters in Eq,(B2) yields:

$$U(t)=E_o-C_i\frac{(E_\nu-E_o)}{(\lambda+\delta)}+\frac{\delta(E_\nu-E_o)}{(\lambda+\delta)}t+D(\frac{C_i+\lambda t}{C_i})^p \quad \text{(B11)}$$

At time t=0 one must have the following relation that identifies D:

$$U(0)=E_o-C_i\frac{(E_\nu-E_o)}{(\lambda+\delta)}+D \quad \text{(B12)}$$

Substitution in Eq,(B11) yields:

$$U(t)=E_o-C_i\frac{(E_\nu-E_o)}{(\lambda+\delta)}+\frac{\delta(E_\nu-E_o)}{(\lambda+\delta)}t+\{U(0)-[E_o-C_i\frac{(E_\nu-E_o)}{(\lambda+\delta)}]\}(\frac{C_i+\lambda t}{C_i})^p \quad \text{(B13)}$$

Grouping terms we obtain:

$$U(t)=U(0)(\frac{C_i+\lambda t}{C_i})^p+[1-(\frac{C_i+\lambda t}{C_i})^p]E_o-C_i\frac{(E_\nu-E_o)}{(\lambda+\delta)}]+\frac{\delta(E_\nu-E_o)}{(\lambda+\delta)}t \quad \text{(B14)}$$

At the end of period n with t = 1, $C_i=C_{n-1}$ and $C_n=C_{n-1}+\lambda$ we obtain:

$$U(n)=U(n-1)(\frac{C_n}{C_{n-1}})^{p_n}+[1-(\frac{C_n}{C_{n-1}})^{p_n}]E_o-C_{n-1}\frac{(E_\nu-E_o)}{(\lambda_n+\delta_n)}]+\frac{\delta_n(E_\nu-E_o)}{(\lambda_n+\delta_n)} \text{(B15)}$$

For simplicity in notation let:

$$(\frac{C_n}{C_{n-1}})^{p_n}=(\frac{C_n}{C_{n-1}})^{-\frac{\delta_n}{\lambda_n}}=\rho_{Un} \quad \text{(B16)}$$

$$(E_\nu-E_o)=E(n)-E(n-1)=\Delta E(n) \text{(B17)}$$

Eq,(B15) becomes:

$$U(n)=\rho_{Un}U(n-1)+[1-\rho_{Un}][E(n-1)-C_{n-1}\frac{\Delta E(n)}{(\lambda_n+\delta_n)}]+\frac{\delta_n\Delta E(n)}{(\lambda_n+\delta_n)} \quad \text{(B18)}$$

More explicitly:

$$U(n)=\rho_{Un}U(n-1)+[1-\rho_{Un}][E(n-1)-C_{n-1}\frac{E(n)-E(n-1)}{(\lambda_n+\delta_n)}]+\frac{\delta_n[E(n)-E(n-1)]}{(\lambda_n+\delta_n)}$$

(B19) and grouping the excitations:

$$U(n)=\rho_{Un}U(n-1)+\frac{1}{(\lambda_n+\delta_n)}[\delta_n-(1-\rho_{Un})C_{n-1}]E(n)$$

$$+\{[1-\rho_{Un}][1+\frac{C_{n-1}}{(\lambda_n+\delta_n)}]-\frac{\delta_n}{(\lambda_n+\delta_n)}\}E(n-1) \quad \text{(B20)}$$

Application of these two solutions for the outflow yields first for the inflow excitation that might be delayed:

$$O(n)=\rho_{On}O(n-1)+[1-\sigma_{On}]\frac{\overline{I}(n)}{\delta_n}+[\sigma_{On}-\rho_{On}]\frac{\overline{I}(n-1)}{\delta_n} \quad \text{(B21)}$$

then for the head in the river cell:

$$O(n)=\rho_{On}O(n-1)+\frac{T_n}{(\lambda_n+\delta_n)}[1-(1-\rho_{On})\frac{C_{n-1}}{\delta_n}]\Delta h_f(n)$$

$$+T_n\{[1-\rho_{On}][\frac{1+\frac{C_{n-1}}{(\lambda_n+\delta_n)}}{\delta_n}]-\frac{1}{(\lambda_n+\delta_n)}\}\Delta h_f(n-1) \quad \text{(B22)}$$

Adding the outflow due to inflow and exchange with the river cell yields:

$$O(n)=\rho_{On}O(n-1)+[1-\sigma_{On}]\frac{\overline{I}(n)}{\delta_n}+[\sigma_{On}-\rho_{On}]\frac{\overline{I}(n-1)}{\delta_n}$$

$$+\frac{1}{(\lambda_n+\delta_n)}[\delta_n-(1-\rho_{On})C_{n-1}]\{\frac{T_n\Delta h_f(n)}{\delta_n}\}$$

$$+[(1-\rho_{On})(1+\frac{C_{n-1}}{\lambda_n+\delta_n})-\frac{\delta_n}{\lambda_n+\delta_n}]\{\frac{T_n\Delta h_f(n-1)}{\delta_n}\} \quad \text{(B23)}$$

Defining the coefficients $\alpha_{On}=[(1-\rho_{On})(1+\frac{C_{n-1}}{\lambda_n+\delta_n})-\frac{\delta_n}{\lambda_n+\delta_n}]$ (B24)

And $\beta_{On}=\frac{1}{(\lambda_n+\delta_n)}[\delta_n-(1-\rho_{On})C_{n-1}]$ (B25)

the writing of the equation simplifies to:

$$O(n)=\rho_{On}O(n-1)+[1-\sigma_{On}]\frac{\overline{I}(n)}{\delta_n}+[\sigma_{On}-\rho_{On}]\frac{\overline{I}(n-1)}{\delta_n}$$

$$+\beta_{On}\frac{T_n}{\delta_n}\Delta h_f(n)+\alpha_{On}\frac{T_n}{\delta_n}\Delta h_f(n-1) \quad \text{(B26)}$$

Solution for head in the river cell

$$C_f\frac{d\Delta h_f}{dt}+\Delta h_f=\gamma_O O+2C_{adj}\Delta h_{adj} \quad \text{(C1)}$$

This is an equation of the form with a constant C:

$$C\frac{dU}{dt}+U=E^O+(E^V-E^O)t \quad \text{(C2)}$$

The solution is of the form: $U(t)=De^{-\frac{t}{C}}+Mt+A$ (C3)

$\frac{dU}{dt}=-\frac{D}{C}e^{-\frac{t}{C}}+M$ Substitution in Eq.(C2) yields:

$C(-\frac{D}{C}e^{-\frac{t}{C}}+M)+De^{-\frac{t}{C}}+Mt+A=E^O+(E^V-E^O)t$ which yields:

$MC+A=E^O$ (C3) $M=E^V-E^O$ (C4) and $A=E^O-C(E^V-E^O)$ (C5)

Substitution in Eq.(C3) yields:

$$U(t)=De^{-\frac{t}{C}}+(E^V-E^O)t+E^O-C(E^V-E^O) \quad \text{(C6)}$$

At time zero then $U(0)=D+E^O-C(E^V-E^O)$ which defines $D=U(0)-[E^O-C(E^V-E^O)]$ (C7)

Substitutionyields:

$U(t)=\{U(0)-[E^O-C(E^V-E^O)]\}e^{-\frac{t}{C}}+(E^V-E^O)t+E^O-C(E^V-E^O)$ or

$$U(t)=U(0)e^{-\frac{t}{C}}+[E^O-C(E^V-E^O)]([1-e^{-\frac{t}{C}})+(E^V-E^O)t \quad \text{(C8)}$$

In particular at time n we have setting: $\rho_{Un} = e^{-\frac{1}{C}}$ (C9)
$$U(n) = \rho_{Un}U(n-1) + \{E(n-1) - C[E(n) - E(n-1)]\}([1-\rho_{Un}) + [E(n) - E(n-1)]$$
(C10)

Grouping the excitation terms we have:

$$U(n) = \rho_{Un}U(n-1) + [1 - C([1-\rho_{Un})]E(n) + [C(1-\rho_{Un}) - \rho_{Un}]E(n-1) \quad \text{(C11)}$$

Application of this result to the head in the river cell equation

$C_f \dfrac{d\Delta h_f}{dt} + \Delta h_f = \gamma_O O + C_{adj}\Delta h_{adj}$ (C1) yields:

$$\Delta h_f(n) = \rho_f \Delta h_f(n-1) + [1 - C_f([1-\rho_f)]\gamma_{On}O(n) + [C_f(1- \rho_f) - \rho_f]\gamma_{On-1}O(n-1)$$

$$+C_{adj}[1 - C_f([1-\rho_f)]\Delta h_{adj}(n) + C_{adj}[C_f(1- \rho_f) - \rho_f]\Delta h_{adj}(n-1) \quad \text{(C12)}$$

For simplifying notation we set:

$\alpha_f = [C_f(1- \rho_f) - \rho_f]$ (C13) and $\beta_f = [1 - C_f([1-\rho_f)]$ (C14) and the equation becomes:

$$\Delta h_f(n) = \rho_f \Delta h_f(n-1) + \beta_f \gamma_{On}O(n) + \alpha_f \gamma_{On-1}O(n-1)$$

$$+\beta_f C_{adj}\Delta h_{adj}(n) + \alpha_f C_{adj}\Delta h_{adj}(n-1) \quad \text{(C15)}$$

Relation between Outflow, Inflow and Head in Adjacent Cells

To obtain that relation one needs to eliminate $\Delta h_f(n)$ from Eq.(B27)

$$O(n) = \rho_{On}O(n-1) + [1-\sigma_{On}]\frac{\bar{I}(n)}{\delta_n} + [\sigma_{On} - \rho_{On}]\frac{\bar{I}(n-1)}{\delta_n}$$

$$\frac{T_n}{(\lambda_n + \delta_n)}\beta_{On}\Delta h_f(n) + \frac{T_n}{(\lambda_n + \delta_n)}\{\alpha_{On} - \beta_{On} + \zeta_{On}]\}\Delta h_f(n-1) \quad \text{(B27)}$$

using the equation

$$\Delta h_f(n) = \rho_f \Delta h_f(n-1) + \beta_f \gamma_{On}O(n) + [\alpha_f - \beta_f]\gamma_{On-1}O(n-1)$$

$$+2C_{adj}[\beta_f \Delta h_{adj}(n) + (\alpha_f - \beta_f)\Delta h_{adj}(n-1)] \quad \text{(C15)}$$

Leading to:

$$O(n)=\rho_{On}O(n-1)+[1-\sigma_{On}]\frac{\overline{I}(n)}{\delta_n}+[\sigma_{On}-\rho_{On}]\frac{\overline{I}(n-1)}{\delta_n}$$

$$+\frac{T_n}{(\lambda_n+\delta_n)}\beta_{On}\{\rho_f\Delta h_f(n-1)+\beta_f\gamma_{On}O(n)+[\alpha_f-\beta_f]\gamma_{On-1}O(n-1)\}$$

$$+\frac{T_n}{(\lambda_n+\delta_n)}[\alpha_{On}-\beta_{On}+\zeta_{On}]\Delta h_f(n-1)$$

$$+\frac{T_n}{(\lambda_n+\delta_n)}2C_{adj}\beta_{On}\{\beta_f\Delta h_{adj}(n)+(\alpha_f-\beta_f)\Delta h_{adj}(n-1)\} \quad \text{(C16)}$$

Grouping terms with each variable one obtains:

$$[1-\frac{T_n}{(\lambda_n+\delta_n)}\beta_{On}\beta_f\gamma_{On}]O(n)=[1-\sigma_{On}]\frac{\overline{I}(n)}{\delta_n}+[\sigma_{On}-\rho_{On}]\frac{\overline{I}(n-1)}{\delta_n}$$

$$+[\rho_{On}+\frac{T_n}{(\lambda_n+\delta_n)}\beta_{On}(\alpha_f-\beta_f)\gamma_{On-1}]O(n-1)$$

$$+\frac{T_n}{(\lambda_n+\delta_n)}[\beta_{On}\rho_f+\alpha_{On}-\beta_{On}+\zeta_{On}]\Delta h_f(n-1)$$

$$+[2\frac{T_n}{(\lambda_n+\delta_n)}C_{adj}\beta_{On}\beta_f]\Delta h_{adj}(n)$$

$$+[2\frac{T_n}{(\lambda_n+\delta_n)}C_{adj}\beta_{On}(\alpha_f-\beta_f)]\Delta h_{adj}(n-1)\} \quad \text{(C17)}$$

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APPENDIX 2. Results for numerical example

routingflow.mpw

DAY	AI	O1	O2	O3	O4	O5	O6	DS1	DS2	DS3	DS4	DS5	DS6
1	80.00	82.90	82.91	84.79	82.71	83.04	0.00	0	00000				
2	83.65	82.59	82.61	83.57	82.56	82.66	0.00	0	00000				
3	94.47	94.07	94.09	95.26	94.03	94.14	0.00	0	00000				
4	112.15	111.88	111.91	113.34	111.85	111.96	0.00	0	00000				
5	136.15	135.82	135.87	137.66	135.80	135.93	0.00	0	00000				
6	165.73	165.22	165.29	167.45	165.21	165.36	0.00	0	00000				
7	200.00	199.22	199.31	201.84	199.23	199.39	0.00	0	00000				
8	237.92	236.82	236.93	239.80	236.86	237.01	0.00	0	00000				
9	278.32	276.91	277.03	280.19	276.98	277.10	0.00	0	00000				
10	320.00	318.31	318.41	321.80	318.39	318.46	0.00	0	00000				
11	278.32	274.76	274.69	276.63	274.86	274.60	0.00	0	00000				
12	237.92	235.39	235.21	236.32	235.47	235.06	0.00	0	00000				
13	200.00	198.28	198.03	198.41	198.36	197.82	0.00	0	00000				
14	165.73	164.69	164.40	164.20	164.78	164.16	0.00	0	00000				
15	136.15	135.67	135.38	134.74	135.77	135.12	0.00	0	00000				
16	112.15	112.14	111.86	110.93	112.26	111.59	0.00	0	00000				
17	106.68	107.43	107.19	106.42	107.55	106.94	0.00	0	00000				
18	101.48	102.18	101.95	101.17	102.31	101.72	0.00	0	00000				
19	96.53	97.22	97.01	96.24	97.35	96.78	0.00	0	00000				
20	91.82	92.49	92.30	91.54	92.62	92.08	0.00	0	00000				
21	87.35	88.08	87.90	87.07	88.22	87.68	0.00	0	00000				
22	83.09	83.96	83.79	82.82	84.11	83.57	0.00	0	00000				
23	79.03	80.03	79.86	78.78	80.18	79.65	0.00	0	00000				
24	75.18	76.27	76.12	74.93	76.44	75.90	0.00	0	00000				
25	71.51	72.69	72.55	71.27	72.86	72.33	0.00	0	00000				
26	68.02	69.27	69.14	67.79	69.45	68.93	0.00	0	00000				
27	64.71	66.01	65.90	64.48	66.20	65.68	0.00	0	00000				
28	61.55	62.91	62.80	61.34	63.10	62.59	0.00	0	00000				
29	58.55	59.95	59.85	58.34	60.14	59.65	0.00	0	00000				
30	55.69	57.12	57.04	55.49	57.32	56.84	0.00	0	00000				
31	52.98	54.43	54.35	52.79	54.63	54.16	0.00	0	00000				
32	50.39	51.87	51.80	50.21	52.06	51.61	0.00	0	00000				
33	47.94	49.42	49.36	47.76	49.62	49.18	0.00	0	00000				
34	45.60	47.09	47.04	45.43	47.29	46.86	0.00	0	00000				
35	43.37	44.87	44.83	43.21	45.07	44.66	0.00	0	00000				
36	41.26	42.76	42.72	41.10	42.96	42.56	0.00	0	00000				
37	39.25	40.74	40.72	39.10	40.94	40.55	0.00	0	00000				
38	37.33	38.82	38.80	37.19	39.02	38.65	0.00	0	00000				
39	35.51	37.00	36.98	35.37	37.19	36.83	0.00	0	00000				
40	33.78	35.25	35.25	33.65	35.45	35.10	0.00	0	00000				
41	32.13	33.60	33.59	32.00	33.79	33.45	0.00	0	00000				
42	30.57	32.02	32.02	30.44	32.20	31.88	0.00	0	00000				
43	29.07	30.51	30.52	28.96	30.70	30.39	0.00	0	00000				
44	27.66	29.08	29.09	27.55	29.26	28.96	0.00	0	00000				
45	26.31	27.71	27.73	26.20	27.90	27.61	0.00	0	00000				
46	25.02	26.41	26.44	24.92	26.59	26.32	0.00	0	00000				
47	23.80	25.18	25.20	23.71	25.35	25.09	0.00	0	00000				
48	22.64	24.00	24.03	22.55	24.17	23.92	0.00	0	00000				
49	21.54	22.88	22.91	21.45	23.05	22.80	0.00	0	00000				
50	20.49	21.81	21.84	20.41	21.98	21.74	0.00	0	00000				
51	19.49	20.79	20.83	19.41	20.96	20.73	0.00	0	00000				
52	18.54	19.82	19.86	18.46	19.99	19.77	0.00	0	00000				
53	17.63	18.90	18.94	17.56	19.06	18.85	0.00	0	00000				
54	16.77	18.02	18.07	16.71	18.18	17.98	0.00	0	00000				
55	15.96	17.19	17.23	15.89	17.34	17.15	0.00	0	00000				
56	15.18	16.39	16.44	15.12	16.54	16.36	0.00	0	00000				
57	14.44	15.63	15.68	14.38	15.78	15.60	0.00	0	00000				
58	13.73	14.91	14.96	13.68	15.06	14.88	0.00	0	00000				

59	13.06	14.23	14.27	13.01	14.37	14.20	0.00	0	00000
60	12.43	13.57	13.62	12.38	13.71	13.55	0.00	0	00000
61	11.82	12.95	13.00	11.78	13.09	12.93	0.00	0	00000
62	11.24	12.36	12.41	11.20	12.49	12.34	0.00	0	00000
63	10.70	11.79	11.84	10.66	11.93	11.78	0.00	0	00000
64	10.17	11.26	11.31	10.14	11.39	11.25	0.00	0	00000
65	9.68	10.74	10.80	9.65	10.87	10.74	0.00	0	00000
66	9.21	10.26	10.31	9.18	10.39	10.25	0.00	0	00000
67	8.76	9.79	9.85	8.73	9.92	9.79	0.00	0	00000
68	8.33	9.35	9.41	8.30	9.48	9.35	0.00	0	00000
69	7.92	8.93	8.99	7.90	9.05	8.94	0.00	0	00000
70	7.54	8.53	8.59	7.52	8.65	8.54	0.00	0	00000

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DAY	H1	H2	H3	H4	H5	H6	DHF1	DHF2	DHF3	DHF4	DHF5	DHF6	DHADJ
1	1.515	1.516	1.536	1.513	1.517	0.000	1.066	1.066	0.964	1.077	1.059	0.000	0.9639
2	1.512	1.512	1.523	1.512	1.513	0.000	1.200	1.196	0.964	1.220	1.180	0.000	0.9639
3	1.635	1.635	1.647	1.634	1.636	0.000	1.304	1.298	0.952	1.331	1.274	0.000	0.0000
4	1.814	1.814	1.828	1.814	1.815	0.000	1.389	1.378	0.903	1.422	1.346	0.000	-2.0000
5	2.038	2.038	2.054	2.038	2.039	0.000	1.473	1.455	0.829	1.512	1.414	0.000	-2.0000
6	2.292	2.293	2.311	2.292	2.293	0.000	1.585	1.559	0.756	1.630	1.507	0.000	-2.0000
7	2.564	2.565	2.585	2.565	2.566	0.000	1.729	1.690	0.683	1.777	1.626	0.000	-2.0000
8	2.845	2.846	2.866	2.845	2.846	0.000	1.904	1.848	0.611	1.953	1.771	0.000	-2.0000
9	3.125	3.126	3.147	3.125	3.126	0.000	2.108	2.032	0.538	2.156	1.941	0.000	-2.0000
10	3.397	3.398	3.419	3.398	3.398	0.000	2.338	2.240	0.467	2.383	2.133	0.000	-2.0000
11	3.110	3.110	3.123	3.111	3.109	0.000	2.535	2.416	0.396	2.575	2.297	0.000	-2.0000
12	2.835	2.833	2.841	2.835	2.832	0.000	2.650	2.522	0.328	2.688	2.397	0.000	-2.0000
13	2.557	2.555	2.558	2.558	2.554	0.000	2.711	2.579	0.264	2.748	2.451	0.000	-2.0000
14	2.288	2.285	2.284	2.288	2.283	0.000	2.725	2.594	0.203	2.761	2.467	0.000	-2.0000
15	2.037	2.034	2.028	2.037	2.032	0.000	2.701	2.575	0.145	2.737	2.452	0.000	-2.0000
16	1.817	1.814	1.805	1.818	1.811	0.000	2.649	2.530	0.090	2.685	2.411	0.000	-2.0000
17	1.770	1.768	1.760	1.772	1.766	0.000	2.589	2.477	0.036	2.624	2.363	0.000	-2.0000
18	1.718	1.716	1.708	1.719	1.713	0.000	2.531	2.426	-0.015	2.566	2.316	0.000	-2.0000
19	1.667	1.665	1.657	1.669	1.663	0.000	2.471	2.373	-0.065	2.507	2.268	0.000	-2.0000
20	1.618	1.616	1.608	1.620	1.614	0.000	2.411	2.319	-0.113	2.447	2.218	0.000	-2.0000
21	1.572	1.570	1.561	1.573	1.567	0.000	2.399	2.313	-0.110	2.435	2.216	0.000	2.0000
22	1.527	1.525	1.515	1.529	1.523	0.000	2.430	2.350	-0.058	2.466	2.257	0.000	2.0000
23	1.484	1.482	1.470	1.485	1.479	0.000	2.453	2.378	-0.007	2.488	2.290	0.000	2.0000
24	1.442	1.440	1.426	1.443	1.437	0.000	2.468	2.399	0.043	2.502	2.316	0.000	2.0000
25	1.401	1.399	1.384	1.403	1.396	0.000	2.478	2.413	0.091	2.510	2.334	0.000	2.0000
26	1.361	1.359	1.343	1.363	1.357	0.000	2.481	2.422	0.137	2.512	2.347	0.000	2.0000
27	1.322	1.320	1.303	1.324	1.318	0.000	2.479	2.425	0.183	2.509	2.355	0.000	2.0000
28	1.284	1.283	1.265	1.286	1.280	0.000	2.473	2.423	0.227	2.501	2.358	0.000	2.0000
29	1.248	1.246	1.227	1.250	1.244	0.000	2.463	2.417	0.270	2.489	2.356	0.000	2.0000
30	1.212	1.211	1.191	1.214	1.208	0.000	2.450	2.407	0.312	2.473	2.351	0.000	2.0000
31	1.177	1.176	1.156	1.180	1.174	0.000	2.434	2.394	0.352	2.455	2.343	0.000	2.0000
32	1.144	1.143	1.122	1.146	1.140	0.000	2.415	2.379	0.392	2.433	2.331	0.000	2.0000
33	1.111	1.110	1.089	1.114	1.108	0.000	2.394	2.361	0.430	2.410	2.317	0.000	2.0000
34	1.079	1.079	1.056	1.082	1.076	0.000	2.371	2.340	0.468	2.385	2.301	0.000	2.0000
35	1.049	1.048	1.025	1.051	1.046	0.000	2.346	2.318	0.504	2.358	2.283	0.000	2.0000
36	1.019	1.018	0.995	1.021	1.016	0.000	2.321	2.294	0.540	2.329	2.263	0.000	2.0000
37	0.990	0.989	0.965	0.992	0.987	0.000	2.294	2.269	0.574	2.299	2.241	0.000	2.0000
38	0.961	0.961	0.937	0.964	0.959	0.000	2.266	2.243	0.608	2.269	2.218	0.000	2.0000
39	0.934	0.934	0.909	0.937	0.931	0.000	2.237	2.216	0.641	2.237	2.194	0.000	2.0000
40	0.907	0.907	0.882	0.910	0.905	0.000	2.207	2.188	0.672	2.206	2.169	0.000	2.0000
41	0.881	0.881	0.856	0.884	0.879	0.000	2.178	2.159	0.704	2.173	2.144	0.000	2.0000
42	0.856	0.856	0.831	0.859	0.854	0.000	2.147	2.130	0.734	2.140	2.117	0.000	2.0000
43	0.832	0.832	0.806	0.835	0.830	0.000	2.117	2.100	0.763	2.108	2.091	0.000	2.0000
44	0.808	0.808	0.782	0.811	0.806	0.000	2.087	2.071	0.792	2.075	2.063	0.000	2.0000
45	0.785	0.786	0.759	0.788	0.783	0.000	2.056	2.041	0.820	2.042	2.036	0.000	2.0000

46	0.763	0.763	0.737	0.766	0.761	0.000	2.026	2.011	0.847	2.009	2.008	0.000	2.0000
47	0.741	0.742	0.715	0.744	0.740	0.000	1.995	1.980	0.874	1.976	1.980	0.000	2.0000
48	0.720	0.721	0.694	0.723	0.719	0.000	1.965	1.950	0.900	1.943	1.952	0.000	2.0000
49	0.700	0.700	0.673	0.703	0.699	0.000	1.935	1.921	0.925	1.911	1.924	0.000	2.0000
50	0.680	0.681	0.654	0.683	0.679	0.000	1.905	1.891	0.950	1.879	1.897	0.000	2.0000
51	0.661	0.662	0.634	0.664	0.660	0.000	1.876	1.861	0.974	1.848	1.869	0.000	2.0000
52	0.642	0.643	0.615	0.645	0.641	0.000	1.847	1.832	0.998	1.817	1.841	0.000	2.0000
53	0.624	0.625	0.597	0.627	0.623	0.000	1.818	1.803	1.020	1.786	1.814	0.000	2.0000
54	0.607	0.607	0.580	0.610	0.606	0.000	1.790	1.775	1.043	1.755	1.787	0.000	2.0000
55	0.590	0.590	0.563	0.593	0.589	0.000	1.762	1.747	1.065	1.726	1.760	0.000	2.0000
56	0.573	0.574	0.546	0.576	0.572	0.000	1.735	1.719	1.086	1.696	1.734	0.000	2.0000
57	0.557	0.558	0.530	0.560	0.556	0.000	1.708	1.691	1.107	1.668	1.708	0.000	2.0000
58	0.541	0.542	0.514	0.545	0.541	0.000	1.681	1.664	1.127	1.639	1.682	0.000	2.0000
59	0.526	0.527	0.499	0.530	0.526	0.000	1.655	1.638	1.147	1.612	1.657	0.000	2.0000
60	0.512	0.513	0.484	0.515	0.511	0.000	1.630	1.612	1.166	1.584	1.632	0.000	2.0000
61	0.497	0.499	0.470	0.501	0.497	0.000	1.605	1.586	1.185	1.558	1.607	0.000	2.0000
62	0.484	0.485	0.456	0.487	0.483	0.000	1.580	1.561	1.203	1.532	1.583	0.000	2.0000
63	0.470	0.472	0.443	0.474	0.470	0.000	1.557	1.537	1.221	1.506	1.559	0.000	2.0000
64	0.457	0.459	0.430	0.461	0.457	0.000	1.533	1.513	1.239	1.481	1.536	0.000	2.0000
65	0.445	0.446	0.417	0.448	0.445	0.000	1.510	1.489	1.256	1.457	1.513	0.000	2.0000
66	0.433	0.434	0.405	0.436	0.432	0.000	1.488	1.466	1.272	1.433	1.491	0.000	2.0000
67	0.421	0.422	0.393	0.424	0.421	0.000	1.466	1.444	1.289	1.410	1.469	0.000	2.0000
68	0.409	0.411	0.381	0.412	0.409	0.000	1.445	1.421	1.305	1.387	1.447	0.000	2.0000
69	0.398	0.400	0.370	0.401	0.398	0.000	1.424	1.400	1.320	1.365	1.426	0.000	2.0000
70	0.387	0.389	0.359	0.391	0.387	0.000	1.403	1.379	1.335	1.343	1.406	0.000	2.0000

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DAY	AI	QS1	QS2	QS3	QS4	QS5	QS6	LC1	LC2	LC3	LC4	LC5	LC6
1	80.000	0.858	0.867	0.000	0.930	0.817	0.000	0.1879	0.1900	0.0000	0.2102	0.1756	0.0000
2	83.646	0.652	0.640	0.000	0.655	0.623	0.000	0.1958	0.1900	0.0000	0.2102	0.1756	0.0000
3	94.474	0.689	0.683	0.000	0.680	0.677	0.000	0.1957	0.1900	0.0000	0.2102	0.1756	0.0000
4	112.154	0.903	0.894	0.000	0.887	0.887	0.000	0.1972	0.1900	0.0000	0.2102	0.1756	0.0000
5	136.149	1.231	1.212	0.000	1.209	1.201	0.000	0.1991	0.1900	0.0000	0.2102	0.1756	0.0000
6	165.731	1.586	1.555	0.000	1.552	1.540	0.000	0.2013	0.1900	0.0000	0.2102	0.1756	0.0000
7	200.000	1.935	1.894	0.000	1.885	1.879	0.000	0.2036	0.1900	0.0000	0.2102	0.1756	0.0000
8	237.915	2.251	2.206	0.000	2.182	2.197	0.000	0.2057	0.1900	0.0000	0.2102	0.1756	0.0000
9	278.324	2.507	2.471	0.000	2.423	2.476	0.000	0.2074	0.1900	0.0000	0.2102	0.1756	0.0000
10	320.000	2.688	2.674	0.000	2.591	2.700	0.000	0.2089	0.1900	0.0000	0.2102	0.1756	0.0000
11	278.324	1.500	1.635	0.000	1.396	1.768	0.000	0.2102	0.1900	0.0000	0.2102	0.1756	0.0000
12	237.915	0.467	0.719	0.000	0.374	0.928	0.000	0.2088	0.1900	0.0000	0.2102	0.1756	0.0000
13	200.000	-0.378	-0.053	0.000	-0.474	0.213	0.000	0.2073	0.1900	0.0000	0.2102	0.1756	0.0000
14	165.731	-1.045	-0.682	0.000	-1.155	-0.376	0.000	0.2057	0.1900	0.0000	0.2102	0.1756	0.0000
15	136.149	-1.539	-1.170	0.000	-1.673	-0.839	0.000	0.2035	0.1900	0.0000	0.2102	0.1756	0.0000
16	112.154	-1.867	-1.516	0.000	-2.031	-1.173	0.000	0.2013	0.1900	0.0000	0.2102	0.1756	0.0000
17	106.684	-1.783	-1.474	0.000	-1.961	-1.147	0.000	0.1991	0.1900	0.0000	0.2102	0.1756	0.0000
18	101.481	-1.759	-1.471	0.000	-1.941	-1.154	0.000	0.1987	0.1900	0.0000	0.2102	0.1756	0.0000
19	96.532	-1.727	-1.459	0.000	-1.912	-1.152	0.000	0.1981	0.1900	0.0000	0.2102	0.1756	0.0000
20	91.824	-1.691	-1.442	0.000	-1.878	-1.145	0.000	0.1975	0.1900	0.0000	0.2102	0.1756	0.0000
21	87.346	-1.753	-1.520	0.000	-1.949	-1.225	0.000	0.1970	0.1900	0.0000	0.2102	0.1756	0.0000
22	83.086	-1.901	-1.679	0.000	-2.111	-1.381	0.000	0.1964	0.1900	0.0000	0.2102	0.1756	0.0000
23	79.034	-2.027	-1.818	0.000	-2.249	-1.519	0.000	0.1959	0.1900	0.0000	0.2102	0.1756	0.0000
24	75.179	-2.133	-1.938	0.000	-2.367	-1.639	0.000	0.1954	0.1900	0.0000	0.2102	0.1756	0.0000
25	71.512	-2.223	-2.042	0.000	-2.467	-1.744	0.000	0.1949	0.1900	0.0000	0.2102	0.1756	0.0000
26	68.025	-2.298	-2.131	0.000	-2.551	-1.836	0.000	0.1943	0.1900	0.0000	0.2102	0.1756	0.0000
27	64.707	-2.360	-2.206	0.000	-2.620	-1.914	0.000	0.1938	0.1900	0.0000	0.2102	0.1756	0.0000
28	61.551	-2.409	-2.270	0.000	-2.676	-1.982	0.000	0.1933	0.1900	0.0000	0.2102	0.1756	0.0000
29	58.549	-2.449	-2.323	0.000	-2.721	-2.040	0.000	0.1928	0.1900	0.0000	0.2102	0.1756	0.0000
30	55.694	-2.479	-2.367	0.000	-2.756	-2.089	0.000	0.1923	0.1900	0.0000	0.2102	0.1756	0.0000
31	52.978	-2.501	-2.402	0.000	-2.782	-2.130	0.000	0.1918	0.1900	0.0000	0.2102	0.1756	0.0000
32	50.394	-2.516	-2.430	0.000	-2.801	-2.163	0.000	0.1913	0.1900	0.0000	0.2102	0.1756	0.0000
33	47.936	-2.525	-2.451	0.000	-2.812	-2.190	0.000	0.1908	0.1900	0.0000	0.2102	0.1756	0.0000
34	45.598	-2.528	-2.466	0.000	-2.817	-2.212	0.000	0.1903	0.1900	0.0000	0.2102	0.1756	0.0000
35	43.375	-2.527	-2.476	0.000	-2.817	-2.228	0.000	0.1898	0.1900	0.0000	0.2102	0.1756	0.0000
36	41.259	-2.521	-2.481	0.000	-2.813	-2.239	0.000	0.1893	0.1900	0.0000	0.2102	0.1756	0.0000
37	39.247	-2.512	-2.482	0.000	-2.804	-2.246	0.000	0.1888	0.1900	0.0000	0.2102	0.1756	0.0000
38	37.333	-2.499	-2.479	0.000	-2.791	-2.250	0.000	0.1883	0.1900	0.0000	0.2102	0.1756	0.0000

39	35.512	-2.484	-2.472	0.000	-2.776	-2.250	0.000	0.1879	0.1900	0.0000	0.2102	0.1756	0.0000
40	33.780	-2.466	-2.463	0.000	-2.757	-2.247	0.000	0.1874	0.1900	0.0000	0.2102	0.1756	0.0000
41	32.133	-2.447	-2.452	0.000	-2.737	-2.242	0.000	0.1869	0.1900	0.0000	0.2102	0.1756	0.0000
42	30.566	-2.426	-2.438	0.000	-2.714	-2.234	0.000	0.1865	0.1900	0.0000	0.2102	0.1756	0.0000
43	29.075	-2.403	-2.423	0.000	-2.690	-2.225	0.000	0.1860	0.1900	0.0000	0.2102	0.1756	0.0000
44	27.657	-2.379	-2.405	0.000	-2.664	-2.213	0.000	0.1855	0.1900	0.0000	0.2102	0.1756	0.0000
45	26.308	-2.354	-2.386	0.000	-2.637	-2.200	0.000	0.1851	0.1900	0.0000	0.2102	0.1756	0.0000
46	25.025	-2.328	-2.367	0.000	-2.609	-2.186	0.000	0.1847	0.1900	0.0000	0.2102	0.1756	0.0000
47	23.804	-2.302	-2.346	0.000	-2.581	-2.170	0.000	0.1842	0.1900	0.0000	0.2102	0.1756	0.0000
48	22.643	-2.275	-2.324	0.000	-2.551	-2.154	0.000	0.1838	0.1900	0.0000	0.2102	0.1756	0.0000
49	21.539	-2.248	-2.301	0.000	-2.521	-2.136	0.000	0.1834	0.1900	0.0000	0.2102	0.1756	0.0000
50	20.489	-2.220	-2.278	0.000	-2.491	-2.118	0.000	0.1829	0.1900	0.0000	0.2102	0.1756	0.0000
51	19.489	-2.193	-2.254	0.000	-2.461	-2.099	0.000	0.1825	0.1900	0.0000	0.2102	0.1756	0.0000
52	18.539	-2.165	-2.230	0.000	-2.431	-2.080	0.000	0.1821	0.1900	0.0000	0.2102	0.1756	0.0000
53	17.635	-2.138	-2.206	0.000	-2.400	-2.060	0.000	0.1817	0.1900	0.0000	0.2102	0.1756	0.0000
54	16.775	-2.110	-2.182	0.000	-2.370	-2.040	0.000	0.1813	0.1900	0.0000	0.2102	0.1756	0.0000
55	15.957	-2.083	-2.157	0.000	-2.339	-2.020	0.000	0.1809	0.1900	0.0000	0.2102	0.1756	0.0000
56	15.178	-2.056	-2.133	0.000	-2.309	-2.000	0.000	0.1805	0.1900	0.0000	0.2102	0.1756	0.0000
57	14.438	-2.029	-2.108	0.000	-2.280	-1.979	0.000	0.1801	0.1900	0.0000	0.2102	0.1756	0.0000
58	13.734	-2.003	-2.084	0.000	-2.250	-1.959	0.000	0.1797	0.1900	0.0000	0.2102	0.1756	0.0000
59	13.064	-1.977	-2.060	0.000	-2.221	-1.938	0.000	0.1794	0.1900	0.0000	0.2102	0.1756	0.0000
60	12.427	-1.951	-2.036	0.000	-2.192	-1.917	0.000	0.1790	0.1900	0.0000	0.2102	0.1756	0.0000
61	11.821	-1.925	-2.012	0.000	-2.164	-1.897	0.000	0.1786	0.1900	0.0000	0.2102	0.1756	0.0000
62	11.244	-1.900	-1.988	0.000	-2.136	-1.877	0.000	0.1783	0.1900	0.0000	0.2102	0.1756	0.0000
63	10.696	-1.876	-1.965	0.000	-2.108	-1.857	0.000	0.1779	0.1900	0.0000	0.2102	0.1756	0.0000
64	10.174	-1.852	-1.942	0.000	-2.081	-1.837	0.000	0.1776	0.1900	0.0000	0.2102	0.1756	0.0000
65	9.678	-1.828	-1.919	0.000	-2.054	-1.817	0.000	0.1772	0.1900	0.0000	0.2102	0.1756	0.0000
66	9.206	-1.805	-1.897	0.000	-2.028	-1.797	0.000	0.1769	0.1900	0.0000	0.2102	0.1756	0.0000
67	8.757	-1.782	-1.875	0.000	-2.003	-1.778	0.000	0.1765	0.1900	0.0000	0.2102	0.1756	0.0000
68	8.330	-1.760	-1.853	0.000	-1.978	-1.759	0.000	0.1762	0.1900	0.0000	0.2102	0.1756	0.0000
69	7.924	-1.739	-1.832	0.000	-1.953	-1.740	0.000	0.1759	0.1900	0.0000	0.2102	0.1756	0.0000
70	7.537	-1.717	-1.811	0.000	-1.929	-1.722	0.000	0.1756	0.1900	0.0000	0.2102	0.1756	0.0000

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