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The Mathematical Knowledge, Sensibilities and Skills in Teaching of Elementary Fraction Concept

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

Article Information

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ABSTRACT

Objective: This research is based on the identification of the mathematical knowledge, sensibilities, and skills of Panama's Elementary Public School teachers who teach fraction concepts and are looking to identify the reasons why the Panamanian students do not show any interest towards mathematics.

Study Design: Facts such as, sensibilities and skills in the mathematical knowledge, could affect the teaching process of the students from elementary school.

The objective of this study is to research qualitative-quantitative factors, with the Likert inquiry and a knowledge test in the area of fractions. The whole analysis is guided by the study of the triangulation methods, people and moments, which is presented and we can use it as a combination of qualitative and quantitative methods (mixed analysis).

Methodology: In this descriptive research was there realized a small exploration in 100 teachers of

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elementary education, split into 32 from first grade teachers, 46 from second grade and 22 of third grade.

Results: In our country, there is a reality about certain shortcomings when our teachers provide to students the fraction concept in the mathematics class. In one of the question of the component about sensibilities, says: I consider mathematics is a subject that can have success easily. The teachers themselves are in disagreement taking mathematics as an issue, and therefore They don't teach when they are faced to the common classes. In this question, 44% of the teachers disagreed, because it is not their strong side. While in the knowledge test about the concept of fractions that mediate their knowledge, the results were surprising since 58% was below the average for basic knowledge of mathematics and the fraction of 42% have the knowledge needed to teach math classes.

Conclusion: This study showed that the mathematical knowledge of the teachers in elementary school is one of the possible causes of the performance of the students in upper levels, leading in this case the students to low performance. Therefore, a good level of comprehension has to be obtained and good knowledge of the concepts of math that could allow them to manage themselves with security and fluency in the topics given to their students and acknowledge the articulation of the purpose and content of the different levels in elementary.

Keywords: Sensibilities; teaching difficulty; skills; knowledge; teacher and capability.

1. INTRODUCTION

The following research is about the mathematical skills and sensibilities of the elementary education teachers of initial cycle 6 to 8 years of the first three levels of primary education of the urban population of Panama. [1] explains the role of the teacher such as fundamental in the teaching process; teachers are one that motivates the desire to learn in the student. Instead of analyzing the curriculum or specify what teacher should think or what they should know, we begin our research by starting, examining and analyzing their practices. There was based the interaction of math and pedagogy in the mathematical teaching at elementary level.

The specific objective is to identify the skills (mathematical understandings) and sensibilities in the affective sense (personal responsibility, understanding students and the perception of mathematical phenomenon) that teachers have in order to teach math classes in the basic level (see Fig. 1). The quality of mathematics teaching depends on teachers' knowledge that the subject should not be a surprise. Yet, he rarely does improvements efforts to consider this into account. Equally unsurprising is that many teachers shortcomings of mathematical understanding and skill. This is unsurprising because teachers are graduates seeking to improve their own opportunities to learn mathematics have been uneven, and often inadequate, just like those of their non-teaching peers.

The analysis were required due to what we know nowadays about the math resources that are necessary for math education, these roles are put into practice and by implication as opportunities for the teachers to learn future development skills with the final objectives to teach math.

In our country, we have seen the lack of interest, the constant failures and the mathematical shortcomings from the students. This is based in the fact that most of the students, at higher middle levels, lack of math basics, Nothing compare to the students who graduate from public schools and to the ones graduated from private schools, which have insufficient knowledge in the field of mathematical reasoning [2]. By the other hand poor results obtained in different test leading to measure the quality of education in which Panamanian students (SERCE and PISA), in the Test SERCE (Second Regional Comparative and Explicative Study), applied to 13 thousand students in elementary, Panama was placed underneath the Latin-American average in almost all the evaluated aspects.

The importance of the capability of a child at a short age to read, communicate, sing, calculate and work with confidence with mathematical ideas, could not be exaggerated [3,4]. The criticism is referred to the identification of better educational practices to allow the students from Elementary Schools to acquire comprehension and appreciation for math, and it is importance in adult life.

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Fig. 1. Operational definition of the study variables

investigation Therefore this work was concentrated in giving answers, among others, questions such, how teachers in elementary levels are prepared to impart math classes? Are all the math contents known by the teachers? Do the teachers only need to know the topics that have to be imparted to the students? From these questions, the present study addresses two things that are related with them: analysis the professional knowledge of the teachers, mathematical knowledge, specifically in a mathematical awareness that is specifically in sensibilities, and skills.

The reason why the topic about "Fractions" was selected for the development of this research came from the curiosity to know more about what was hidden behind the concept of them, since half of the high school students have failed on learn how to prioritize in elementary levels because of the Education Ministry. Therefore, we detected learning gaps that should be filled in this level and have students start assimilating previous schemes. Nevertheless, we have also found errors that came from teaching this concept by teachers.

This work is born by to know one of the pillars of the didactic triangle, the teacher that in order to be able to achieve an appropriate learning in the students; it depends on the teacher performance and motivation in your classroom. To start delimiting this object sets the didactic system in three places: T- teach, S-student and K-knowledge [5].

2. MATERIALS AND METHODS

The study of professional knowledge of the teachers and the effects of it in students is a topic of interest in the education area, added up to the sensibilities that come from the moment to prepare classes. The literature allusion is made [6] to the role of the teacher as to identify different ways or styles of learning, looking to modify or design homework sequences that are real problems and that allow students to think about the importance of math.

Some researches about the professional knowledge of teachers has identified different knowledge that a teacher should have to develop in their career [7,8,9]. In Panama, mathematical knowledge, skills and sensibilities of the teachers have been little studied, is important to concentrate more from this topic. Because the success any improvement in the quality of education in our country depends on this features.

In order to study the mathematical knowledge of the teachers we need to go deeply about the objective of our study, regarding to the fraction concept. In the Panamanian syllabus, this concept is exposed as a topic that allows students to relate quantities expanded to the numeration system and make use of the new symbols system for its representation. The content is treated to allow students to learn a meaningful concept thought and the use of the different models for the fractions study [10].

It's here where the teacher should focus on looking for a method in which math is taught in order to motivate the student to achieve how to practice difficulties. But to do this you must be prepared, and be aware of the group learning styles, besides, it should not be so confident in what he or she takes in their way of thinking, in which they have learned when they were students. The reflexive way to prepare what he will teach and confront daily errors that took the corrections that are requested, remembering that we are facing children with longing for learning and for doing mathematics.

Morales and García [2] makes a careful follow up to the progress and difficulties of the students in class and mentions that they depend in the capability and disposition of all the students to use the mathematical language to express their ideas. Giving to the students a place to provide good question and to be able to elaborate conjectures with the intention to solve their problems. In this sense, the process of conjectures like an element of construction to a mathematical thought for the students, most of the time is something that couldn't fostered the teacher in class leaving behind the previous knowledge that the students have [11].

This article does not pretend to shade the vision of mathematical knowledge in Panama, can be a tool to design patterns required and improve the way of teaching in the early years of education, but in turn, and why not, try to improve the path of teaching in general.

2.1 Methods

The whole analysis is guided by the root cause of this study, its triangulation (methods, people and moments) is presented and used by a combination of qualitative and quantitative methods (mixed analysis); this study has two variables to measure in the mathematical knowledge. The first is the sensitivity supported with a Likert scale, with questions concerning liability, understanding and perception and second, the teacher's skills. This was done with a test on mathematical fraction concept assessed by experts. The main point was to give information about the teacher's skills and sensibilities in West Panama's Elementary Schools.

Parrilla et al. [12] affirms that the descriptive studies questions things and pretend to inform about a topic or scope based on studies about itself. The development of this investigation was made under a not experimental design, at on where categories of analysis were not manipulated, given the nature of the focus of the investigation. It is a transcendental type, due that study was concentrated in a defined period in which [13], consists to observe phenomenon the way there are given in their natural context, to later analysis and identify them.

In the first part of the investigation, survey statements were used to reflect different ways of thinking about mathematics, the mathematical knowledge and the skills to do math. In which we use three of the components, the affective (the emotion / the feeling), the cognitive (the knowledge and know) and behavioral (the behavior). Pretending the objective of the study, the investigation was made from a qualitative focus point, quantitative and interpretative. Starting from the quantitative variable that we have observe with the values from the restrained variables, there are no intermediate values in between two specific values. From the ordinal quantitative variable, we cite in order each category but they are not represented numerical. In addition, from the numerical variable, it was allowed to interpret in a meaningful way the results where surveys were made.

During this first phase, a scale-Likert survey was applied to the teachers that teach math in elementary level. Later, a tabulation that allowed to gather each guestion results (scale 15-items), were made concerning liability, understanding and perception. Based on the survey the results on the sensibilities from math teachers, in which it was able to depict what could they do to transmit appropriate mathematical the knowledge to their students failing to provide a good mathematical learning. Consequences may result in not to teach students the mathematic requirements, according to the educational level.

Before we start the first phase of the study an instrument was built for Panamanian teachers; at

the beginning of the research, according to the scaling technique proposed by Likert, which represent an actual focus and a very popular scale to measure the variable that concluded the study. This had a structure of 5-point Likert scale and 15-item scale, each one was codified according to its address (positive or negative). The options presented were totally agree, in agreement, undecided, not in agreement and in total disagreement. In order to teach these options the following score were assigned:

Table 1.	Allocation	of the	points to	o the items
	in the	Likert	scale	

Option	Score with reagents positive direction	Score with reagents with negative direction
Totally agree	5	1
Agree	4	2
Undecided	3	3
In	2	4
disagreement		
Total	1	5
disagreement		

It was applied to 100 teachers taken to safflower. To validate the instrument obtained there was an index of validity and reliability of 0.677 this indicates that the instrument is reliable and that the results will be consistent and coherent. In the second stage a knowledge test was applied to teachers with a series of fraction problems, the objective is to measure their knowledge in mathematical competences in elementary (understand, interpret, quantify, analyze, relate, solve, decide, etc.). Obtaining a reliability of 0.801 in the written test with scores from 0 to 8 in the development evaluated by experienced judges.

For this investigation a sample of 100 teachers from elementary level were distributed into 32 teachers from first grade teachers, 46 teachers from second grade and 22 teachers from third grade that impart classes in the West part of Panama; from which 82 were women and 18 were men. The age range went from 25 to 58 years old. This investigation was started at the middle of the academics period 2014. The instrument was applied on a schedule in which the teachers handed down their classes. They were informed that were answering a questionnaire in order that they would offer his answers without fear or be worried since it would not affect them in their job field. The teachers took approximately 15 minutes to answer the 15 items and for the second part of the test time, wasn't measured, in order for them to answer calmly and this way specifically in their level of agreement or disagreement on a symmetric agree-disagree scale for a series of statements.

3. RESULTS AND DISCUSSION

The scales show, that the development of the math classes were affected negatively, the fundamental tools that a teacher uses, plays an important role in the knowledge given to the student, it prevents from not giving the class due to the lack of knowledge in certain concept and shows the multiple results of the investigation. To begin, it is necessary to have a clear understanding of what types of population is being handled; because of them, the details of the variables are shown, starting from gender. Table 2 shows the distribution of teachers by gender and age, this allows us to show a minimum age of 25 years old, and a maximum age of 58 years old and a middle age of 42 years old. The fashion age was 40 years old with a typical deviation of 8.61 respectively.

Table 2. Frequency distribution by genderand age of the study sample

	Age range	Gender	
		Men	Woman
	25-34 years old	6	15
	35-44 years old	4	37
Age	45-54 years old	6	21
	55-65 years old	2	9
	Total sample	18	82

Regarding to the analysis results achieved in the instruments of mathematical knowledge, as it was indicated in its elaboration, which consisted in the distribution of organizing two frequent ways, the sensibilities and skill teacher. With the total sample, it was found that, teachers show concentration on points 1 and 2, which shows that these people have sensibilities to be negative or partially negative. The middle range value of the sample for the total sample of the study was 2.07 points (s= 0.596). Attending to the math sensibilities and skills, its knowledge, teaching, could infer that these teachers reject the discipline.

In one of the question about sensibilities components it's quote: I consider mathematics is

a subject that can have success easily. This question was considerate to each teacher in respect to their mathematic development versus the group; therefore, not to teach when they face the common classes; in which 44% answered to be in total disagreement and to be distinguished in this subject (is not their strength), a 9% is undecided now of answering and 3% agrees now of distinguishing in math.

Table 3. Distribution Percentage in the scale
points of the instrument sensibilities in math
applied to the sample

Points	Percentage (%)
1	11
2	73
3	10
4	6
Total	100

Moreover, the knowledge test on concept in fraction; evaluated by judges, consisted in 8 problems of mathematical competences with fractional content, every problem has a weighting of one point. The results are startling since 58% was under the middle range in basic mathematical knowledge of fraction and one 42% have the necessary skills to impart classes.

Some teachers ensure to lack elements (mathematical contents) to teach math, they admit that in occasion it's difficult for them transfer knowledge because they do not know the contents and the depths of these, mainly topics such as: concept of number. fraction. percentage or rational numbers. We can see it in the following table: Is observed as the mathematical knowledge is contrasted with the sensitivity and skill of teachers. Is answered one of research questions, teachers have not built the concept of fractions, so that your knowledge are incomplete and teaching that can give, is poor, whereby the educational implications in the results found were determined for elementary students.

The fact that teachers are at this level of deficiency in knowledge of fractional the structure, will prevent promote the true construction the fraction concept in the student, since it will focus on results and not the process. Moreover, not knowing that there is a process of building the concept of fractions, the teacher does not detect the process step on which each student stands in order to achieve advancement in this.

Paradoxically the teacher of primary school in Panama has large spaces dedicated to teaching several subjects. He claims he does not know about all procedures to construct mathematical knowledge and mathematical content, therefore it is not easy will help build a favorable meaning of mathematics for him nor he can easily make the transfer of the mathematics content for child.

Table 4. Descripti	ve statistics of teacher
score on skills in I	mathematical test in the
fraction co	ncept knowledge

	Sensibilities	Mathematical knowledge
Data		
Mean	1.97	4.68
Standard	0.397	1.89
deviation		
Variance	0.158	3.57
Asymmetry	1.236	-0.853
Kurtosis	2.43	-0.072
Mín.	1	0
Máx.	4	8

With the above results sensitizes us to the fact that students in the Bachelor of Primary Education curriculum guidelines should be in which mathematics learning should be meaningful and to get students should learn mathematics with understanding, actively building new knowledge from experience and prior knowledge [14], and they well may transmit it to their students.

The purpose of a teacher in the classroom is to treat their students develop mathematical reasoning, formulate and solve problems. Moreover, this teaching should pay attention to the organization of teaching, careful selection of the examples and teaching tasks or situations that provide opportunities for students to investigate significant problems for them. There is a contrast in item 4 on advance planning your class, because they are aware that some students have managed to learn dynamically [15].

Teachers also manifest, which as you can teach what you do not understand enough, dependence developed by teachers in their previous knowledge is evident lack of autonomous strategies for teaching fractions. Rigidly limited to the use of textbooks, and it's almost no input into the strategy instruction undertake mechanically. We have to take into account that not all results were negative, important aspects can be highlighted, such as:

- Be aware that if they energize the math class the students will learn better.
- Know that they have to dedicate time to the math class as well as the other subjects.
- Invest part of their salary to buy materials to motivate their students in class.
- Try to assist to seminars to improve constantly.

The following graph shows the correlation skill vs sensibilities is moderate with a score of 0.564, $\rho = 0.01$. Is observed that teachers with low sensitivity has poor mathematical ability, while those who are concerned about their profession (sensitivity) have more ability, for reasons of perfecting that they try in order to motivate his students. Recall that these teachers must teach five different subjects in a day and sometimes they avoid giving at math class for fear of being wrong, implanted in the students conceptual "holes".

3.1 Discussion

The results obtained showed a great difference and shows the point of view that the teachers have. Nevertheless, on the other hand, it also teaches us how to manage the knowledge that the Panamanian teachers have, which is crucial in the students learning process, this leads us to think that an appropriate knowledge and good management of mathematical skills are crucial to have a logic and appropriate organization, to impart math classes.

The problem on low mathematical performance has nothing to do with "teachers not teaching correctly" instead; it is more related to the methods learned during their education practice and from where it is believed traditionally that students are learning.

It is based fundamentally, in comprehension and acceptance of the traditional focus, in which math is being thought in Panama, which has not been developed the competences of the students of basics in mathematics needed.



Fig. 2. Correlation of the variables skills and sensibilities

The Panamanian Educational Legislature establishes that the Ministry of Education and Public Universities (Law 47 from 1946, Organic of education, modified by law 34 of 1995) must attend the teaching formation, together, but in the latest results obtained with SERCE noticed the weakness in higher education [16]. Nevertheless, teaching formation has been evolving in the last years, has shown significant progress, and due to the establishment of Education by competences could have a meaningful meaning in students learning. Just as shown:

The mathematical knowledge in a person is a tendency to respond to mathematical polemic situations, by reflexing on problems and the solutions in a social context and the construction or reconstruction of actions, process and objects organizing them in diagrams to treat this situation [17]. Taking this into account we can mention, that it is necessary that teachers that impart classes in crucial stages, must dominate or have knowledge from where part origin of the mathematical knowledge and dominate topics such as fractions.

The usual solution is to require teachers to study more mathematics. Many additional propose coursework for teachers (Panama Ministry of Education), and some argue that elementary teachers should be specialists in mathematics. Nevertheless. increasing the quantity of mathematics' teacher's coursework, will only improve the quality of mathematics teaching method, if teachers learn mathematics in certain ways that make a difference for the skill with which they are able to do their work and the sensibilities in understanding students. The goal is not to produce teachers who know more mathematics. but it will improve the Panamanians students learning methods.

Is pointed out that not only the knowledge influences the mathematical interest. Must be broken this paradigm, since motivation and dynamics with to teach classes, target the child to accept positively or negatively the mathematical teaching, ie prevents these students see this subject as a barrier, and shall establish interest for to know about math and will help promote a positive environment in problem solving.

In the controversy with results the test skill the Panamanian teachers than considered have no basic knowledge in mathematics. Was demonstrated on test, which indicate the deficits in mathematical skills, which leads us to believe that could be lack of interest of the teacher in the classroom, this one is not aware of the degree of impact that can be caused to his pupil, which assists its class and depends on what teachers develop.

The teacher should have in mind that there are many factors that influence the acceptance of math in children, and one of these elemental factors and most important is the affective or emotional factors.

4. CONCLUSION

The knowledge and skills identified in teaching math are relevant by this time; we are talking about a rate of teachers who are not very well prepared on teaching math, so this situation is very scary, because education is the main component to develop a country, also it is the main element of combat social problems within society. Yet another factor influencing this problem is the fact that students suffer from not an adequate family integration, with households where parents work outside home the whole day and spend enough time with their children in order to support them and supervise their homework [4].

On the other hand, we can see the lack of interest from teachers who are not involved in the process of pedagogical innovation, through different moments of training. They have not acquire a commitment to enhance the teaching process in math (concept of fraction) and in this way to help the development of mathematical competences in their students also to improve their learning.

It is not enough with only thinking that everything is great, it is necessary to be aware the we are teaching math to children from elementary level, which is the basis of education. We all know that a building that does not have a solid base sooner or later will have consequences; from school dropouts, poor performances knowledge and all this leads to a worrying social problem.

The little interest that teachers show towards math and the lack of dynamism, knowledge and skills to teach their classes is one of the causes of poor performance and they play with the children instability. A poor performance of a teacher, leads us to a deep reality in which we should take a quick action, to support to teaching staff in the development of professional skills that make it more effective to lead the learning math process in their students.

It is necessary that the Ministry of Education draws up a comprehensive assessment nationwide for mathematical knowledge at the same time create a campaign of evaluation of math and to realize policy of awareness of the damage caused in the performance of mathematics. Create workshops in every school at the national level to help change the negative aptitude, which is observed in the education and establish a more favourable climate for teachers and students.

ETHICAL APPROVAL

All authors hereby declare that all experiments have been examined and approved by the appropriate ethics committee and have therefore been performed in accordance with the ethical standards laid down in the 1964 declaration of Helsinki.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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