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New Approach for the Calculation of Critical Depth in a U-Shaped Channel

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Authors' contributions

This work was carried out in collaboration between both authors. Author BA designed the study, carried out the literature search, wrote the draft of the manuscript. Author MLN wrote the final version of the article. Both authors read and approved the final manuscript.

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ABSTRACT

The U-Shaped channel is a high performance structure widely used in practice. Currently, the problem of the critical depth in such profile does not have a direct solution. Current methods of calculation are based on complex mathematical procedures or optimal fitting methods, often generating unacceptable errors in practice, knowing that the calculation of the critical depth requires a high accuracy. The complexity of the problem stems from the fact that the flow governing equations are complicated due to the shape of the profile. In this study, the form of the flow equation is simple through the intake of the properties of the triangle. Furthermore, even if the equation is implicit, its resolution is possible by applying the fixed-point method with an initial value judiciously chosen. The process converges after the seventh step of calculating only and leads to an almost exact solution. A calculation example is presented that highlights the simplicity of the calculation procedure.

Keywords: U-Shaped channel; critical depth; fixed-point method; discharge; hydraulics.

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1. INTRODUCTION

Critical depth plays a major role in determining the subcritical or supercritical nature of the flow and in the classification of varied flow [1-3]. In this regard, several studies have been devoted to the calculation of critical depth in various geometric profiles of channels [4-13], offering solutions with varying degrees of accuracy. Several methods were used to calculate the critical depth such that numerical method, graphical method, explicit regression-based equations, fitting curves and Newton-Raphson iterative method. Regarding the U-channel, a relatively recent study has been devoted to the calculation of the critical depth using fitting curves [14]. The relative error caused by this method is about 0.7%, which can be an important relative error for a certain number of practical cases. The calculation of critical depth requires a much smaller relative error. The methods of calculating the critical depth in the U-Shaped channel are complex due to the geometrical shape of the channel. The U-Shaped channel may be considered as a triangular channel with a rounded bottom. Using the properties of a triangle, one obtains equations of simple form such that the critical water area and the critical top width that come into consideration in the criterion for critical flow. The form of the equation of critical flow is handy, contrary to that obtained in previous studies.

This equation is implied towards the non-dimensional critical depth. However, its form is so simple that one can apply standard methods of resolution, such as the fixed-point method. This is the method which is used in the present study, with a suitable initial value. The iterative process is not constraining since almost exact solution is obtained after the seventh step of calculation. A practical example is provided showing both the high accuracy and the simplicity of the calculation.

2. CRITICAL FLOW EQUATION

Fig. 1 shows the critical flow in a U-Shaped channel. The critical depth is y_c and the critical top width is T_c, where the subscript "*c*" denotes the condition of the critical state of the flow. The channel is characterized by the side slope *m* horizontal to 1 vertical. The vertical linear dimension *y* is the depth of the flow in the hypothetical triangle obtained by extending the sides of the channel.

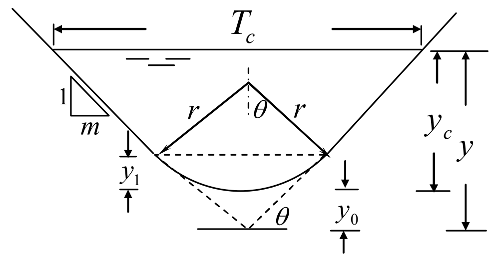

Fig. 1. Critical flow in a U-Shaped channel

The critical top width T_c can be written as:

$$T_c = 2my \tag{1}$$

Where $m = cotg\theta$.

The critical water area is expressed as:

$$A_c = mr^2\left[(y/r)^2 - \chi_1\right] \tag{2}$$

Where:

$$\chi_1 = \frac{1}{m}\left(\frac{1}{m} - \sin^{-1}\frac{1}{\sqrt{1+m^2}}\right) \tag{3}$$

The well known criterion for critical flow states that:

$$\frac{Q^2 T_c}{gA_c^3} = 1 \tag{4}$$

Inserting Eq. (1) and Eq. (2) into Eq. (4) leads to:

$$\frac{2myQ^2}{gm^3r^6\left[(y/r)^2 - \chi_1\right]^3} = 1 \tag{5}$$

Eq. (5) can be rewritten as:

$$\frac{2Q^2(y/r)}{gm^2r^5\left[(y/r)^2 - \chi_1\right]^3} = 1 \tag{6}$$

Let us assume the relative conductivity Q^* as:

$$Q^* = \frac{\sqrt{2}Q}{\sqrt{gm^2r^5}} \tag{7}$$

Assume also the following aspect ratio:

$$\eta = y / r \tag{8}$$

Thus, Eq. (6) is reduced to:

$$Q^{*2} = \frac{(\eta^2 - \chi_1)^3}{\eta} \tag{9}$$

Adopt the following change in variables:

$$z = \eta^2 - \chi_1 \tag{10}$$

Eq. (9) can be then simply written as:

$$Q^{*2} = \frac{z^3}{\sqrt{z + \chi_1}} \tag{11}$$

Or else:

$$z = Q^{*2/3}(z + \chi_1)^{1/6} \tag{12}$$

Eq. (12) is implicit towards the variable *z*. To solve Eq. (12) we suggest a numerical method which consists in approaching successively the solution. The calculation process is iterative and operates on Eq. (12) after selecting a first value of *z*. Assume that the first value of *z* is $z_o = \chi_1$. As a result, the next values of *z* are obtained such that:

$$z_1 = Q^{*2/3}(2\chi_1)^{1/6} \tag{13}$$

$$z_2 = Q^{*2/3}(z_1 + \chi_1)^{1/6} \ldots\text{and so on} \tag{14}$$

The calculation process stops when z_i and z_{i+1} are sufficiently close. It is obvious that the speed of convergence of the described iterative process depends strongly on the value of z_o initially selected. With $z_o = \chi_1$, intensive calculations showed that almost exact value of *z* is obtained, in the worst case, at the end of the seventh step of calculating only. The suggested procedure of calculation is not therefore constraining.

We tested 1120 numerical examples which showed that the proposed iterative method converges. We did not test all cases that may arise in practice in studying the convergence of the recommended method. Like any iterative method, the advocated method may not converge.

Once the final value of *z* is determined, the aspect ratio *η* is worked out from Eq. (10) as:

$$\eta = \sqrt{z + \chi_1} \tag{15}$$

The non-dimensional critical depth can be expressed as:

$$\eta_c = \eta - y_0 / r \tag{16}$$

Where:

$$y_0 / r = \frac{\sqrt{1 + m^2}}{m} - 1 \tag{17}$$

Thus:

$$\eta_c = \sqrt{z + \chi_1} - \frac{\sqrt{1 + m^2}}{m} + 1 \tag{18}$$

Considering Eq. (8), one can obtain the critical depth sought as:

$$y_c = r\eta_c \tag{19}$$

3. PRACTICAL EXAMPLE

Compute the critical depth y_c in the U-Shaped channel shown in Fig. 1 for the following data:

Q = 10 m^3/s, m = 1 (θ = 45°), r = 0.8m

(For the sake of calculation, the counts will not be rounded off)

1. According to Eq. (7), the relative conductivity Q^* is:

$$Q^* = \frac{\sqrt{2}Q}{\sqrt{gm^2r^5}} = \frac{\sqrt{2} \times 10}{\sqrt{9.81 \times 1^2 \times 0.8^5}} = 7.8877934$$

2. Compute $z_0 = \chi_1$ using Eq. (3). Hence:

$$\chi_1 = \frac{1}{m}\left(\frac{1}{m} - \sin^{-1}\frac{1}{\sqrt{1 + m^2}}\right)$$
$$= \frac{1}{1} \times \left(\frac{1}{1} - \sin^{-1}\frac{1}{\sqrt{1 + 1^2}}\right) = 0.21460184$$

One can show that in the case of this example, the recommended iterative method converges on one hand and converges at the end of the seventh step of calculation on the other hand. To show the convergence of the method, it suffices to check that the first derivative of the function to the right of Eq. (12) is less than unity for z_1. We have:

$$F(z) = Q^{*2/3}(z+\chi_1)^{1/6}$$

Thus:

$$F'(z) = \frac{1}{6}\frac{Q^{*2/3}}{(z+\chi_1)^{5/6}}$$

Compute for:

$$z_1 = Q^{*2/3}(z_0+\chi_1)^{1/6} = 7.8877934^{2/3} \times (0.21460184 + 0.21460184)^{1/6} = 3.44149854$$

Then:

$$F'(z_1) = \frac{1}{6}\frac{Q^{*2/3}}{(z_1+\chi_1)^{5/6}} = \frac{1}{6} \times \frac{7.8877934^{2/3}}{(3.44149854 + 0.21460184)^{5/6}} = 0.2242009$$

As we can see, the first derivative is less than unity, which confirms that the method converges.

We can even calculate the number of iterations needed to solve the problem. For this, consider an absolute error and an interval for z. Choose a relative error such that $\varepsilon = 10^{-5}$ and an interval for z such that $[a;b] = [0.21460184\,;6]$. This is an extremely wide range that encompasses most practical cases. The value 0.21460184 corresponds to $z_0 = \chi_1$. The number of iterations *n* is expressed by the following relationship:

$$n \geq \frac{\ln\left(\dfrac{\varepsilon}{b-a}\right)}{\ln\left[F'(b)\right]}$$

The calculation leads to:

$$F'(b) = F'(6) = \frac{1}{6} \times \frac{7.8877934^{2/3}}{(6+0.21460184)^{5/6}} = 0.14409249$$

Thus:

$$n \geq \frac{\ln\left(\dfrac{10^{-5}}{6-0.21460184}\right)}{\ln\left[0.14409249\right]} = 6.84884305 \cong 7$$

The iterative process converges after the seventh step of calculating.

Inserting the obtained values of Q^* and χ_1 in Eq. (12) and adopting the described iterative process for $z_0 = \chi_1$, the final value of *z* is such that:

$$z_6 \approx z_7 = z = 5.260588152$$

3. According to Eq. (15), the aspect ratio η is as:

$$\eta = \sqrt{z + \chi_1} = \sqrt{5.260588152 + 0.21460184} = 2.339912389$$

4. Consequently, the non-dimensional critical depth η_c is worked out from Eq. (18) as:

$$\eta_c = \sqrt{z + \chi_1} - \frac{\sqrt{1+m^2}}{m} + 1 = 2.339912389 - \frac{\sqrt{1+1^2}}{1} + 1 = 1.925698827$$

5. Finally, the required critical depth y_c is given by Eq. (19) as:

$$y_c = r\eta_c = 0.8 \times 1.925698827 = 1.54m$$

6. This step aims to verify the criterion for critical flow governed by Eq. (4). The critical top width T_c is given by Eq. (1) as:

$$T_c = 2my = 2mr(y/r) = 2mr\eta$$

Hence:

$$T_c = 2mr\eta = 2 \times 1 \times 0.8 \times 2.339912389 = 3.743859822m$$

According to Eq. (2), the critical water area A_c is:

$$A_c = mr^2\left[(y/r)^2 - \chi_1\right] = mr^2(\eta^2 - \chi_1)$$

Thus:

$$A_c = mr^2(\eta^2 - \chi_1) = 1 \times 0.8^2 \times (2.339912389^2 - 0.21460184) = 3.366776417m^2$$

Inserting the values of Q, A_c and T_c in Eq. (4) results in:

$$\frac{Q^2 T_c}{g A_c^3} = \frac{10^2 \times 3.743859822}{9.81 \times 3.366776417^3} = 1.000017829 \approx 1$$

Thus, the criterion for critical flow is verified confirming the validity of the calculations.

4. CONCLUSIONS

The study has been devoted to the calculation of critical depth in a U-Shaped channel. The Hydraulic parameters of the flow, such as the critical top width and the critical water area, were deducted from the geometric properties of a triangle. Consequently, the criterion of critical flow has led to an implicit equation of simple form on which has been applied the fixed point method with an initial value judiciously chosen. The purpose of this resolution was to determine the aspect ratio of the hypothetical triangle. This parameter was closely related to the non-dimensional critical depth by a simple explicit relationship. A calculation example was provided to explain the procedure for computing the critical depth. The advocated iterative process was not constraining since the solution was obtained after the seventh step of calculation only. The last step of calculation has successfully verified the criterion of critical flow, thereby confirming the validity of the calculations.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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