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A Financial Prey-predator Model with Infection in the Predator

Letetia Mary Addison[1], Balswaroop Bhatt[1*] and David Owen[1]

[1]Department of Mathematics and Statistics, Faculty of Science and Technology, The University of the West Indies, St. Augustine Campus, Trinidad and Tobago.

Authors' contributions

This work was carried out in collaboration among all authors. Author LMA designed the mathematical model, performed the analyses, managed the literature searches and wrote the first draft of the manuscript. Authors BB and DO approved the design of the model, the analyses in the study and literature searches. All authors read and approved the final manuscript.

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Abstract

A modified predator-prey model is proposed with logistic growth in both prey and predator populations and an infection in the predator population. This model uses ideas from the original biological Lotka-Volterra model in an attempt to imitate financial predation. Potential investors, the prey, interact with financial experts, the predators, who provide advice on purchasing financial instruments and investments. Some of these experts are honest while others are 'infected', in that, a portion of them attempt to deceive clients into making irrational investments for their own benefit, incurring losses to the client. Stability and Hopf bifurcation analyses are discussed analytically. The results have been verified using numerical simulations in MATLAB. Using different datasets, the study shows that variation of different parameters can affect the stability of the system and the co-existence of potential investors and financial experts over time.

Aims: To determine regions of stability for the model by varying parameter values in simulated datasets.

Study Design: Stability Analysis – Analytical and Numerical.

Methodology: The model is first studied analytically by solving for the positive, interior equilibrium point (once it exists) when the differential equations are solved simultaneously. Stability conditions are defined based on these results using the Routh – Hurwitz criteria. These results are verified numerically

*Corresponding author: E-mail: Bal.Bhatt@sta.uwi.edu;

in MATLAB as an initial value problem, integrating with slight perturbation around the equilibrium point. Each parameter in the dataset is varied one at a time. The regions where the model remains stable for a particular parameter are recorded. The Hopf bifurcation points, where there are stability changes, are also noted and graphical simulations are produced using time series and three-dimensional (3D) plots for particular stable and unstable scenarios. The effect of three different pairs of financial expert - investor persuasion rates on the populations of investors, is also shown.

Results: Hopf bifurcation parameters for three simulated datasets were found to be associated with the investment persuasion rate, investor-expert interaction rates, expert interaction rate, expert interference constant and growth rate of experts. In particular, the population of potential investors increased drastically when the investor persuasion rate for dishonest investors was much greater than that for honest investors for the given dataset.

Conclusion: Variation of parameter values in the model allow for the analysis of model stability conditions. These enhance the creation of detection mechanisms to control the 'infection' of investment fraud among financial experts.

Keywords: Prey-predator; financial predation; fraud; stability; bifurcations.

1 Introduction

Financial predation is analogous to the traditional biological idea in which the predator attacks a prey for its food source. This gravely affects businesses, which operate in a similar way to biological eco-systems. According to Bricage [1], the two are very similar, in that, they both exhibit complex, non-linear, dynamic characteristics. Abraham-Froi [2] noted that the cyclic behaviour of economy is explained by the financial structure of investment, while Wilcox [3] maintained that the dynamics in investment markets are analogous to organisms competing for information on prices. Hence, ecological ideas can assist in the partial understanding of the complex dynamics involved in the financial eco-system.

In the financial world, a predator has the ability to prey on finances in order to gain profit from investments. In many cases, they do so dishonestly. The predators are invariably multi-million, if not multi-billion dollar companies, and the prey represents the average investor or financial institutions in charge of groups of average investors, noted by Mesly [4]. The Great Depression of 1929 and the financial recession crisis in 2008 have both inspired financial experts and mathematicians alike, to study models which can assist in prediction of such anomaly events.

There are a variety of factors which lead to financial anomalies, including mismanagement of funds, as well as fraud. Hellwig [5] indicated that financial crises are caused by lack of proper risk assessment on the part of financial institutions. In addition to this, Bergeron and Laroche [6] noted that 'the ideas of perceived predation enables these financial players to hide their agendas, since they assume that a negative image leads to increased perceived risk.' This is the technique used by some financial experts to mislead and therefore gain the trust of potential investors.

Consequently, individual and corporate investors alike, can be victims of financial fraud. The social interactions of key players in the financial world are similar to prey and predatory interactions in Biology. Mesly [7] discussed this idea of perceived predation in the context of finance, noting that some key players are willing to take advantage of unsuspecting investors. Subsequently, this causes these individuals unforeseen financial losses. This motivates the behaviour of financial experts: they can choose to evaluate the market honestly or to be blatantly dishonest in their assessments. Some may even do both, in different situations.

As a result, different mathematical methods have been employed to measure this form of predation. Mesly [4] used factorial analysis, cluster analysis, structural equation modelling and multiple regression techniques to identify psychological factors, such as trust and wealth, and how these affect the predation process. These variables are useful in testing the behavioural effects of these predators on their victims. A predation coefficient is also presented as an indication of a potential investor's perceived deception by a financial agent.

Mesly [8] extended these notions in another model in order to determine whether financial predators can be detected before they commit fraud. The four resource elements: essential, nonessential, work and knowledge, were used to generate a predatory curve to detect abnormal behaviours. These provided a possible indication of the presence of financial predators. The implication for the average or wealthy investor is obvious: detecting the predator (before he can act) may mean saving thousands if not millions of dollars, also noted by Mesly [8].

Consequently, Mesly [9] investigated this idea further, indicating that there is a neurobiological inclination for financial players to behave as either predators or prey. They have the ability to choose different prey or predator approaches. They may choose to maintain their initial investment, withdraw from the market, or take advantage of vulnerable 'prey' consumers. A similar idea was explored in Lim et al. [10], where behavioural finance and consumer behavior was expressed in an investment intentions model. This used structural equation modelling and factors such as perceived risk, product knowledge and uncertainty to make conclusions about individual decision-making.

Although these methods have been used to detect predation, financial fraud has continued to plague the industry in the form of investment schemes. In 2008, the chairman of the NASDAQ Stock Exchange, Bernard Madoff, led a multibillion-dollar Ponzi scheme, originally developed by Charles Ponzi in 1920. Artzrouni [11] and Zhu et al. [12] discussed the principle behind the operation: it lured potential investors using promises of high rates of returns and the scheme spreads via word-of-mouth. Rantala [13] compared the growth of Ponzi schemes to the 'spread of infectious diseases' and Shiller [14] proposed that these infectious social dynamics tend to inflate investment prices.

The notion that investment ideas can be spread socially has foundations in the theory from infectious disease models within a population. Anderson and May [15] were instrumental in formulating a predator-prey model with infection of the prey species. Their work was based on the original Lotka - Volterra [16,17] model for competing species, in addition to the model for infectious disease transmission originally studied by Kermack and Mc Kendrick [18]. The latter model was known as the SIR (Susceptible-Infected-Removed) compartmental model. The idea here was that a species contains individuals susceptible to a disease, infected with the disease and those who had recovered or died from the infection. Over the years, there have been variations to the original form.

Although Chattopadhyay and Arino [19], Han et al. [20], Xiao and Chen [21], Pada Das [22], Sahoo and Poria [23], Greenhalghet al. [24], Kant and Kumar [25], have explored the effects of predation on disease models, the application of this particular modified model to financial predation is a fairly new area. There is very little work done using the Lotka- Volterra predator – prey model, with the presence of infection, to assess the stability of the financial eco-system with the "disease" of financial fraud. This idea is one which motivates our study. Therefore, it is worth exploring this notion further.

Biologically, prey populations influence the size of predator populations and vice versa. Wilcox [3] also discussed the link between biological predator-prey models and the stock market as an eco-system, with the idea that the dynamics of the former assists in the understanding of the stability of the market. The analogy between investment approaches and species makes it clear that one does not want the analogies to famines and parasitic diseases caused by overpopulation within a particular investment approach (Wilcox [3]). This is also a motivating factor in our study.

In this work, potential investors are the financial representation of prey. Financial experts, the predators, are divided into two classes: honest and dishonest. Unlike in biology, entrepreneurs can choose whether to enter as predator or prey (Mehlum et al. [26]). Using this idea, financial experts can decide whether to be honest or dishonest. Dishonest experts are 'diseased' and involved in fraudulent schemes, which enables them to deceive potential investors. This is analogous to an ‘infection’ according to biological epidemic models. These experts interact amongst themselves spreading the ‘infection’ where possible to other experts. The dynamics of these interactions, as well as those potential investors, are investigated mathematically using a modified prey-predator model.

Therefore, the goal of this paper is to assess the stability of the model with respect to the number of potential investors and financial experts. In this way, one can find the optimal parameters in which they either co-exist or become ‘extinct’ in investment situations. Analysis of attractor types and the transitions between them as significant model parameters change, that is, the model’s bifurcations, is helpful in understanding stability shifts (Colon et al. [27]). Thus, this work is not meant to replace existing financial or social models, but is a way of showing a novel approach to analyzing these behaviours using stability analysis.

Hence, this paper first provides a brief introduction to the different models in Section 1. Section 2 describes the model with a brief overview of the model assumptions, methodology with an outline of the analytical analysis for obtaining the positive interior equilibrium point. Next, the stability of this point and Hopf bifurcation is examined. Then, in section 3 numerical simulations are provided with graphical interpretations of the results, as well as a discussion of the findings. Finally, section 4 outlines conclusions, future work and recommendations based on this work.

2 Methodology

The model used is a modified version of the original Lotka - Volterra model. Potential investors and financial experts represent the prey (X) and predator (Y) populations respectively. Financial experts describes brokers, agents, bankers or any relevant persons involved in the investment industry. They are divided into susceptible (Y_1) and infected (Y_2) groups. The infection here is the “disease” of fraud toward the potential investors. The model takes the form of a modified version of the prey-predator model proposed by Haque [28], in which there is a transmissible disease among the predator population.

Anderson and May [15] first studied the effect of an epidemic on predation via a modification of Lotka-Volterra prey-predator model with infected prey. Hassell and Varley [29] proposed a trophic function with a functional response of the form:

$$s(X,Y) = \frac{\alpha X}{Y^{\sigma}} \tag{2.1}$$

Where α is the searching efficiency of the predator to find prey and σ represents the interference of the predator (Hassley -Valley constant). Costner et al. [30] discussed the value of this constant, assuming $\sigma \in [\frac{1}{2}, 1)$, realistically, depending on predators forming a fixed number of groups. If predators do not form groups the functional response is that of a ratio – dependent model.

The form of the trophic function used in our research is:

$$r(X) = \frac{\alpha X}{1 + \beta X^{\sigma}} \tag{2.2}$$

This is a prey-dependent functional response. This term, βX^{σ}, is motivated by an Agent-Based Model containing the function for a simplified economy consisting of a goods and credit market (Arslan et al. [31]). The function for the level of a firm’s production (L) is a function of the total capital (K) in the business cycle. It is given by:

$$L = \tau K^{\sigma} \tag{2.3}$$

Where $\tau > 1$ and $0 < \sigma < 1$. In our model, we assume τ can take any positive value and that σ can take a value of 1, as previously discussed in the original ratio-dependent form.

Let X be the money invested by potential investors (prey),

Let Y_1 be the money invested by financial experts (susceptible predators) and

Let Y_2 be the money invested by dishonest financial experts (infected predators involved in controlled fraud as the infection).

The system of differential equations to represent this model is as follows:

$$\begin{aligned}\frac{dX}{dt} &= f(X) - a_1 r_1(X) Y_1 - a_2 r_2(X) Y_2 \\ \frac{dY_1}{dt} &= g_1(Y_1) + a_1 r_1(X) Y_1 - c\varphi_1\,(Y_1, Y_2) \\ \frac{dY_2}{dt} &= g_2(Y_2) + a_2 r_2(X) Y_2 + c\varphi_2(Y_1, Y_2)\end{aligned} \tag{2.4}$$

where

$$f(X) = rX\left(1 - \frac{X}{M}\right), \quad g_i(Y_i) = n_i Y_i\left(1 - \frac{Y_i}{K}\right), \qquad i = 1,2$$
$$r_i(X) = \frac{X}{1 + b_i X^{\sigma_i}}, \quad \varphi_i(Y_1, Y_2) = Y_1 Y_2, \qquad i = 1,2$$

and $a_1, a_2, b_1, b_2, c, K, M, n_1, n_2, r$ are assumed to be positive constants

The system becomes:

$$\begin{aligned}\frac{dX}{dt} &= rX\left(1 - \frac{X}{M}\right) - \frac{a_1 X Y_1}{1 + b_1 X^{\sigma_1}} - \frac{a_2 X Y_2}{1 + b_2 X^{\sigma_2}} \\ \frac{dY_1}{dt} &= n_1 Y_1\left(1 - \frac{Y_1}{K}\right) + \frac{a_1 X Y_1}{1 + b_1 X^{\sigma_1}} - cY_1 Y_2 \\ \frac{dY_2}{dt} &= n_2 Y_2\left(1 - \frac{Y_2}{K}\right) + \frac{a_2 X Y_2}{1 + b_2 X^{\sigma_2}} + cY_1 Y_2\end{aligned} \tag{2.5}$$

Table 1. Definitions of parameters in the model

Parameter	Definition
r	Maximal growth rate of potential investors
M	Carrying capacity of potential investors
K	Carrying capacity of financial experts
a_1	Interaction rate of honest financial experts with potential investors
a_2	Interaction rate of dishonest financial experts with potential investors
b_1	Honest expert -investor persuasion rate
b_2	Dishonest expert - investor persuasion rate
σ_1	Honest financial expert interference constant
σ_2	Dishonest financial expert interference constant
n_1	Maximal growth rate of honest financial experts
n_2	Maximal growth rate of dishonest financial experts
c	Interaction rate of honest and dishonest financial experts

Model Assumptions:

i) A specific population is assumed to grow logistically (potential investors, honest financial experts, dishonest financial experts) in the absence of the other two populations.
ii) Disease among the financial experts is the infection “control fraud” which is accounting fraud used to deceive potential financial investors.
iii) The disease model here is the S-I (Susceptible – Infectious) Model where there is no recovery from the infection.
iv) Incidence rate, c, is the contact rate for every fraudulent financial expert with an honest one and is of bilinear mass action.
v) All populations of investors and experts grow at a logistic rate with carrying capacities M and K respectively.
vi) The honest financial experts interact with the potential investors with functional response, $r_i(X) = \frac{X}{1+b_i X^{\sigma_i}}$, $i = 1,2$ but they may become infected by the dishonest investors.
vii) The infection is assumed to spread among financial experts only at a rate γ by the mass action law.
viii) The total predator population at time t is $Y(t) = Y_1(t) + Y_2(t)$
ix) Total number of potential investors, X, is greater than the total number of financial experts, Y
x) In contrast to the assumption by Pada Das [22], which states that the infected predator is less able to hunt or to capture a prey than a susceptible predator, we assume that the honest and dishonest investors have the ability to use the same effort to attract potential investors. Dishonest investors may be more desperate to attract potential investors.
xi) For simplicity, assume the model contains minimal noise, that is, very little stochastic effects.

A dimensionless system is created for each of computations. The following new variables and parameters are introduced:

$$\tilde{t} = rt, \quad \tilde{x} = \frac{X}{M}, \quad \tilde{y}_1 = \frac{Y_1}{K}, \quad \tilde{y}_2 = \frac{Y_2}{K}, \quad \alpha_1 = \frac{a_1 K}{r}, \quad \alpha_2 = \frac{a_2 K}{r},$$

$$c_1 = b_1 M^{\sigma_1}, \quad c_2 = b_2 M^{\sigma_2}, \quad \rho_1 = \frac{n_1}{r}, \quad \rho_2 = \frac{n_2}{r}, \quad \gamma = \frac{cK}{r}$$

For simplicity, the bar (~) is dropped in the substitutions and the original system in equation (2.5) becomes:

$$\begin{aligned}
\frac{dx}{dt} &= x(1-x) - \frac{\alpha_1 x y_1}{1+c_1 x^{\sigma_1}} - \frac{\alpha_2 x y_2}{1+c_2 x^{\sigma_2}} \\
\frac{dy_1}{dt} &= \rho_1 y_1 (1-y_1) + \frac{\alpha_1 x y_1}{1+c_1 x^{\sigma_1}} - \gamma y_1 y_2 \\
\frac{dy_2}{dt} &= \rho_2 y_2 (1-y_2) + \frac{\alpha_2 x y_2}{1+c_2 x^{\sigma_2}} + \gamma y_1 y_2
\end{aligned} \tag{2.6}$$

The initial condition, $(x(0), y_1(0), y_2(0))$, is the positive interior equilibrium point with slight perturbation.

2.1 Existence of positive, interior equilibrium point

The interior equilibrium point is the non-zero, positive solution obtained using the method outlined in Dubey and Upadhyay [32]. There exists two intersecting isoclines are found at a unique point, (y_1, y_2). Firstly, put $\frac{dx}{dt} = \frac{dy_1}{dt} = \frac{dy_2}{dt} = 0$ where x, y_1 and y_2 all not equal to zero. Let the equilibrium point be (x^*, y_1^*, y_2^*). For simplicity, the asterisk (*) is dropped in calculations.

This gives:

$$x(1-x)-\frac{\alpha_1 x y_1}{1+c_1 x^{\sigma_1}}-\frac{\alpha_2 x y_2}{1+c_2 x^{\sigma_2}}=0 \tag{2.7}$$

$$\rho_1 y_1(1-y_1)+\frac{\alpha_1 x y_1}{1+c_1 x^{\sigma_1}}-\gamma y_1 y_2=0 \tag{2.8}$$

$$\rho_2 y_2(1-y_2)+\frac{\alpha_2 x y_2}{1+c_2 x^{\sigma_2}}+\gamma y_1 y_2=0 \tag{2.9}$$

Using equations (2.8) and (2.9):

$$\frac{\alpha_1 x y_1}{1+c_1 x^{\sigma_1}}=\gamma y_1 y_2-\rho_1 y_1(1-y_1) \tag{2.10}$$

$$\frac{\alpha_2 x y_2}{1+c_2 x^{\sigma_2}}=-\gamma y_1 y_2-\rho_2 y_2(1-y_2) \tag{2.11}$$

Equations (2.10) and (2.11) into (2.7) give:

$$x(1-x)+\rho_1 y_1(1-y_1)+\rho_2 y_2(1-y_2)=0 \tag{2.12}$$

$$x=\frac{1}{2}\pm\sqrt{1+4\rho_1 y_1(1-y_1)+4\rho_2 y_2(1-y_2)} \tag{2.13}$$

For simplicity in calculations, we take the value of $\sigma_i=1,\ i=1{,}2$

From (2.10):

$$x=\frac{(\gamma y_2-\rho_1(1-y_1)}{\alpha_1-c_1(\gamma y_2-\rho_1(1-y_1))} \tag{2.14}$$

From (2.11):

$$x=\frac{-\big(\gamma y_1+\rho_2(1-y_2)\big)}{\alpha_2+c_2\big(\gamma y_1+\rho_2(1-y_2)\big)} \tag{2.15}$$

For x to be positive, the following pairs of conditions must hold in (2.14) and (2.15) respectively:

$$\gamma y_2-\rho_1(1-y_1)>0 \tag{2.16a}$$

$$\alpha_1-c_1\big(\gamma y_2-\rho_1(1-y_1)\big)>0 \tag{2.16b}$$

$$\gamma y_1+\rho_2(1-y_2)<0 \tag{2.17a}$$

$$\alpha_2+c_2\big(\gamma y_1+\rho_2(1-y_2)\big)>0 \tag{2.17b}$$

Solving equations (2.14) and (2.15) give:

$$f(y_1, y_2) \equiv \frac{1}{c_1}\left[\frac{\alpha_1}{\alpha_1 - c_1\big(\gamma y_2 - \rho_1(1 - y_1)\big)} - 1\right] + \frac{1}{c_2}\left[\frac{\alpha_2}{\alpha_2 + c_2\big(\gamma y_1 + \rho_2(1 - y_2)\big)} + 1\right] = 0 \qquad (2.18)$$

Using equations (2.12) and (2.14) gives:

$$g(y_1, y_2) \equiv \frac{1}{c_1}\left[\frac{\alpha_1}{\alpha_1 - c_1\big(\gamma y_2 - \rho_1(1 - y_1)\big)} - 1\right]\left(1 - \frac{1}{c_1}\left[\frac{\alpha_1}{\alpha_1 - c_1\big(\gamma y_2 - \rho_1(1 - y_1)\big)} - 1\right]\right) + \rho_1 y_1(1 - y_1) + \rho_2 y_2(1 - y_2) = 0 \qquad (2.19)$$

From equation (2.18), when $y_2 \to 0$, then $y_1 \to y_{1a}$ and this is defined as the positive solution to the equation:

$$\frac{1}{c_1}\left[\frac{\alpha_1}{\alpha_1 + c_1\big(\rho_1(1 - y_1)\big)} - 1\right] + \frac{1}{c_2}\left[\frac{\alpha_2}{\alpha_2 + c_2(\gamma y_1 + \rho_2)} + 1\right] = 0 \qquad (2.20)$$

Similarly, from equation (2.19), when $y_1 \to 0$, then $y_2 \to y_{2a}$ and this is defined as the positive solution to the equation:

$$\frac{1}{c_1}\left[\frac{\alpha_1}{\alpha_1 - c_1(\gamma y_2 - \rho_1)} - 1\right]\left(1 - \frac{1}{c_1}\left[\frac{\alpha_1}{\alpha_1 - c_1(\gamma y_2 - \rho_1)} - 1\right]\right) + \rho_2 y_2(1 - y_2) = 0 \qquad (2.21)$$

The two isoclines in equations (2.18) and (2.19) intersect at a unique point (y_1, y_2). Using these values, the positive value of x can be calculated from equation (2.13). This completes the existence of the positive equilibrium point.

2.2 Local stability analysis

The stability of the equilibrium point (x, y_1, y_2) is examined, where x, y_1, y_2 are all positive, by finding the Jacobian Matrix. First, the original system of equations in (2.5) is linearized via the following substitutions:

$$x = x + u,$$

$$y_1 = y_1 + v, \qquad (2.22)$$

$$y_2 = y_2 + w,$$

where u, v and w are small perturbations about the equilibrium point. All terms are expanded about the equilibrium point using Taylor's theorem, neglecting higher order terms of u, v, w. The characteristic polynomial takes the form:

$$\lambda^3 + b_2\lambda^2 + b_1\lambda + b_0 = 0 \qquad (2.23)$$

where

$$b_2 = -(j_{11} + j_{22} + j_{33}),$$

$$b_1 = j_{22}j_{11} - j_{12}j_{21} + j_{33}(j_{11} + j_{22}) - j_{31}j_{13} - j_{32}j_{23}), \qquad (2.24)$$

$$b_0 = -j_{11}j_{22}j_{33} + j_{21}j_{12}j_{33} - j_{12}j_{31}j_{23} + j_{11}j_{32}j_{23} - j_{13}j_{21}j_{32} + j_{13}j_{31},$$

and

$$j_{11} = 1 - 2x - \frac{\alpha_1 y_1}{1 + c_1 x^{\sigma_1}} + \frac{\alpha_1 y_1 c_1 x^{\sigma_1} \sigma_1}{(1 + c_1 x^{\sigma_1})^2} - \frac{\alpha_2 y_2}{1 + c_2 x^{\sigma_2}} + \frac{\alpha_2 y_2 c_2 x^{\sigma_2} \sigma_2}{(1 + c_2 x^{\sigma_2})^2}$$

$$j_{12} = -\frac{\alpha_1 x}{1 + c_1 x^{\sigma_1}}$$

$$j_{13} = -\frac{\alpha_2 x}{1 + c_2 x^{\sigma_2}}$$

$$j_{21} = \frac{\alpha_1 y_1}{1 + c_1 x^{\sigma_1}} - \frac{\alpha_1 y_1 c_1 x^{\sigma_1} \sigma_1}{(1 + c_1 x^{\sigma_1})^2} \tag{2.25}$$

$$j_{22} = \rho_1(1 - 2y_1) + \frac{\alpha_1 x}{1+c_1 x^{\sigma_1}} - \gamma y_2$$

$$j_{23} = -\gamma y_1$$

$$j_{31} = \frac{\alpha_2 y_2}{1 + c_2 x^{\sigma_2}} - \frac{\alpha_2 y_2 c_2 x^{\sigma_2} \sigma_2}{(1 + c_2 x^{\sigma_2})^2}$$

$$j_{32} = \gamma y_2$$

$$j_{32} = \rho_2(1 - 2y_2) + \frac{\alpha_2 x}{1 + c_2 x^{\sigma_2}} + \gamma y_1$$

For stability, the eigenvalues, λ, in the characteristic polynomial must have negative real parts. The Routh-Hurwitz criteria provides the conditions to satisfy a stable equilibrium which occurs if and only if

$$b_2 > 0, \ b_0 > 0, \quad b_1 b_2 - b_0 > 0 \tag{2.26}$$

Theorem 2.1. Given an equilibrium point (x, y_1, y_2) satisfying the equations of system (2.5), then once Lemma holds and $j_{11}, j_{12}, j_{13}, j_{21}, j_{22}, j_{23}, j_{31}, j_{32}, j_{33}$ are defined by equation (2.25), then the equilibrium point exists and is locally stable if and only if (2.26) holds where b_0, b_1 and b_2 have been defined in (2.24).

2.3 Existence of Hopf bifurcation(s)

The criterion for Hopf Bifurcation is outlined according to Liu [33]. Consider the system of equations defined in (2.5) which can be written in the general form:

$$\dot{X} = f_n(X), \qquad X \in \mathbb{R}^3, \ \mu \in \mathbb{R}, \ f \in C^\infty \tag{2.27}$$

with the equilibrium point, X_0, at some parameter, $\mu = \mu_0$ defined as (X_0, μ_0).

Consider the coefficients of the characteristic polynomial, b_i, defined in equation (2.23), where $b_i \equiv b_i(\mu), i = 0,1,2$ are smooth functions of μ. In order for a Hopf bifurcation to exist, the following conditions are assumed for the system at a parameter $\mu = \mu_0$:

(i) $$b_0(\mu_0) > 0, \quad b_1(\mu_0) > 0, \quad b_1(\mu_0) b_2(\mu_0) - b_0(\mu_0) = 0 \tag{2.28}$$

and

(ii) $$\frac{d\big(b_1(\mu_0) b_2(\mu_0) - b_0(\mu_0)\big)}{d\mu} \neq 0 \tag{2.29}$$

Theorem 2.2. Given an equilibrium point (x, y_1, y_2) satisfying the equations of system (2.5) satisfying Theorem 2.3, the conditions for the existence of a Hopf Bifurcation are satisfied by equations (2.28) and (2.29).

Due to the complicated nature of the expressions of the three dimensional system (2.5), this analysis is verified numerically.

3 Results and Discussion

Different parameter values are used in the system in order to investigate stability changes in the model. We investigate if the investors and experts can co-exist in terms a steady state solution (a node or a stable focus) or a stable oscillatory solution (limit cycle). The Hopf bifurcation indicates the start or end of a periodic solution from an equilibrium point, when a parameter surpasses a particular value.

Three datasets are shown in Table 2. For each dataset, each parameter was varied individually while keeping the others constant. The results were recorded where there was a change in stability for specific parameters. Dataset (1) had equal values of c_1 and c_2, while in dataset (2) and dataset (3), these values were unequal.

Stable regions and unstable regions occurred on different sides of the Hopf Bifurcation (HB) point(s). This point(s) were found both analytically using the Routh-Hurwitz Criteria. Then, the automated continuation package, MATCONT (version matcont3p4), available in MATLAB [34] was used to verify the bifurcation points numerically.

Table 2. Parameter values for three datasets

Parameter	Dataset (1)	Dataset (2)	Dataset (3)
$\boldsymbol{\alpha_1}$	5.5	5.5	5.5
α_2	0.5	0.5	0.5
c_1	*0.01*	*0.01*	*5*
c_2	*0.01*	*5*	*0.01*
σ_1	1	1	1
σ_2	1	1	1
ρ_1	0.05	0.05	0.05
ρ_2	0.05	0.05	0.05
γ	0.09	0.09	0.09
Equilibrium point	(0.012, 0.067, 1.239)	(0.012, 0.106, 1.271)	(0.027, 0.086, 1.431)
Stability via Theorem 2.1	Stable equilibrium	Unstable equilibrium	Stable equilibrium

Table 2 shows the stability regions and Hopf bifurcation points for the three datasets where the value of c_1 is varied while the other parameters are held constant. In dataset (1), the initial values for c_1 and c_2 are the equal while in datasets (2) and (3), the value of $c_1 \ll c_2$ and $c_1 \gg c_2$ respectively. Financially, these two parameters represent the rate at which financial experts (honest and dishonest) encourage the investors to make an investment (that is, the investor persuasion rate).

In the first dataset, Hopf bifurcations occur for the parameter c_1 only. Figs. 1 and 3 show time series graphs for a stable and unstable case for dataset (1), with respect to the variation of c_1. The stable case for the model at $c_1 = 0.01$ shows a slight fluctuation in the three populations initially. However, eventually over time the populations co-exist and remain stable. The unstable case at $c_1 = 7.5$ shows the continuous fluctuation of all three populations over time.

The other two datasets display similar results for additional parameters. In dataset (2), bifurcations occur at parameters α_1, α_2 and γ. These represent the investor-expert interaction rates, α_i, $i = 1,2$ and the interaction rate between the experts, γ. Dataset (3) has bifurcations for the parameters α_1, σ_1 and ρ_2. These are the rates of investor-honest expert interaction, α_1, the investor-honest expert interference constant, σ_1, as well as the maximal growth rate of the dishonest financial experts, ρ_2. Similar time series graphs can also be plotted to analyse the behaviour of the model for the variations of these parameters.

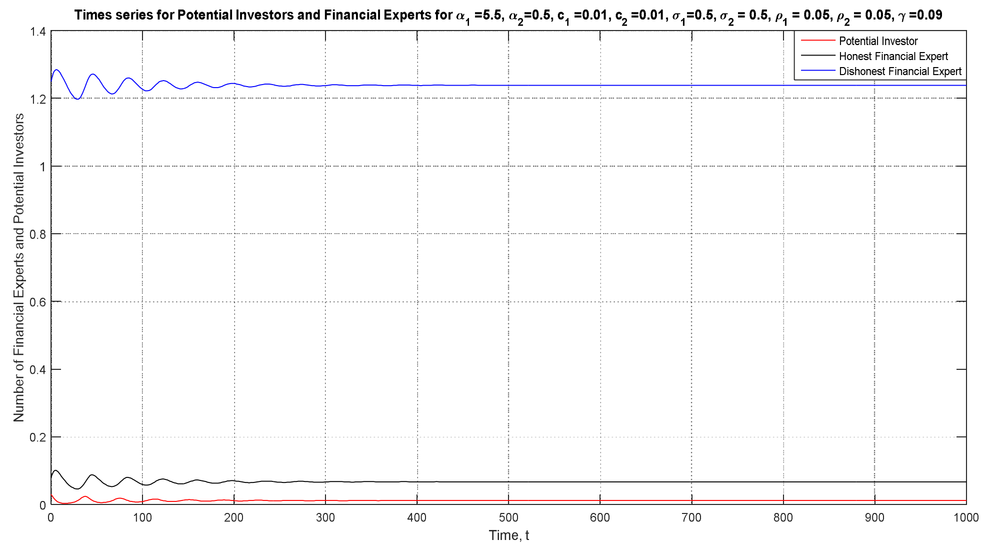

Fig. 1. Time series graph showing stable equilibrium where the populations for Investors and Financial experts co-exist over time for dataset (1) where $c_1 = 0.01$

Figs. 2 and 4 show the behaviour of the orbits of the system close the equilibrium points corresponding to these stable and unstable cases respectively. These graphical simulations enhance the recorded numerical results, since they display how the variations of different parameters over time affect the model stability. Fig. 5 also displays the changes in the population of potential investors with time, for the three pairs of values of financial expert persuasion rates, c_i for $\text{i} = 1, 2$, for datasets (1), (2) and (3) respectively.

Table 3. Stability regions for parameters in datasets (1), (2) and (3)

Dataset	Parameter	Stable (S) /Unstable (U) regions	Hopf bifurcation points
(1)	c_1	$0 < c_1 \leq 6.26$ (S)	6.26
		$6.26 < c_1 \leq 34.09$(U)	34.09
		$c_1 > 34.09$ (S)	
(2)	α_1	$0 < \alpha_1 \leq 0.69$ (S)	0.69
		$\alpha_1 > 0.69$ (U)	
	α_2	$0 < \alpha_2 \leq 0.67$ (S)	0.67
		$0.67 < \alpha_2 \leq 0.82$ (U)	0.82
		$\alpha_2 > 0.83$ (S)	
	γ	$0 < \gamma < 0.07$ (S)	0.07
		$0.07 \leq \gamma \leq 0.69$ (U)	0.69
		$\gamma > 0.69$ (S)	
(3)	α_1	$0 < \alpha_1 \leq 12.88$ (S)	12.88
		$\alpha_1 > 12.88$ (U)	
	σ_1	$0 < \sigma_1 \leq 0.51$ (S)	0.51
		$0.51 < \sigma_1 \leq 0.93$ (U)	0.93
		$\sigma_1 > 0.93$ (S)	
	ρ_2	$0 < \rho_2 \leq 0.37$ (S)	0.37
		$\rho_2 > 0.37$ (U)	

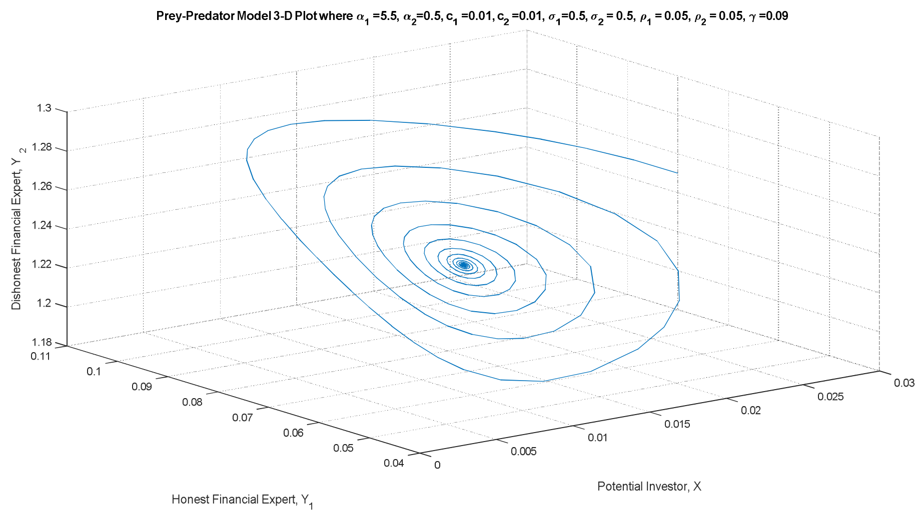

Fig. 2. Stable equilibrium for dataset (1) where a small disturbance from the equilibrium initially eventually re-approaches the equilibrium value again for the populations of investors and experts

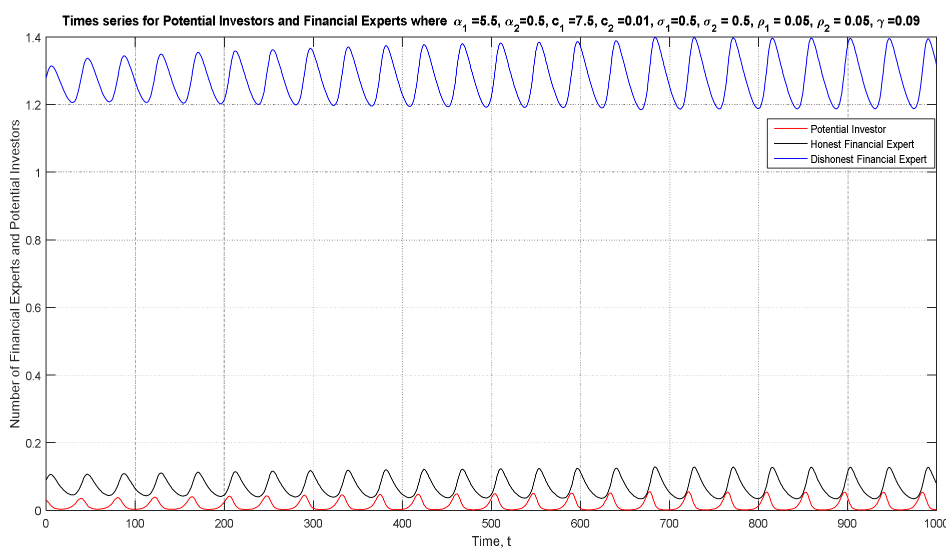

Fig. 3. Time series graph showing unstable equilibrium for the populations of Potential investors and Financial experts over time for dataset (1) where $c_1 = 7.5$

Interesting behavior occurs in dataset (2) where $c_1 = 0.01, c_2 = 5$. The number of potential investors increases drastically when the investor persuasion rate for dishonest experts is much greater than that for honest experts. Here, potential investors are influenced more so by fraudulent experts to enter investment schemes, thereby causing their populations to increase drastically in comparison to datasets (1) and (3) where the dishonest expert persuasion rates are lower.

As a result, a number of reasons may be adequate to partially explain some of the population fluctuations. Financial experts use different tactics such as social media, word-of-mouth and other advertising means to attract their clients. This may cause the co-existence equilibrium of the investors and experts to fluctuate. There are alternating periods of time where investors are drawn to investment schemes or shy away from them due to prior knowledge of dishonest tactics.

Some of these tactics include attractive investments which promise high interest rates and returns. As the influence of the agents and brokers increases, the investors become more educated and are more prone to being careful about risky investments and so the number of potential investors falls. Hence, further analysis of such a model and its parameters can increase the knowledge of how these social behaviours affects the number of players in the market.

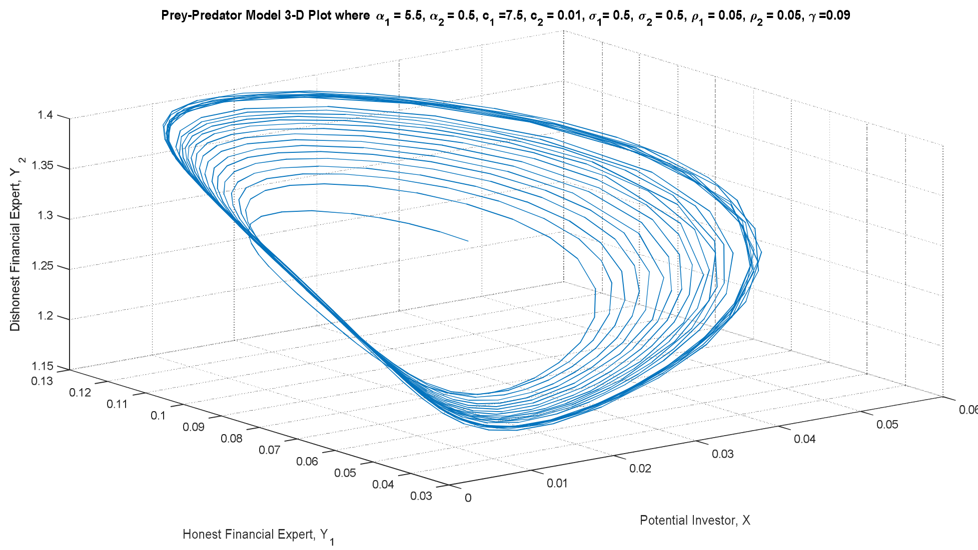

Fig. 4. Unstable equilibrium for dataset (1) where $c_1 = 7.5$ where a small disturbance from the equilibrium value of investors and experts cycles away from this point in an oscillatory manner

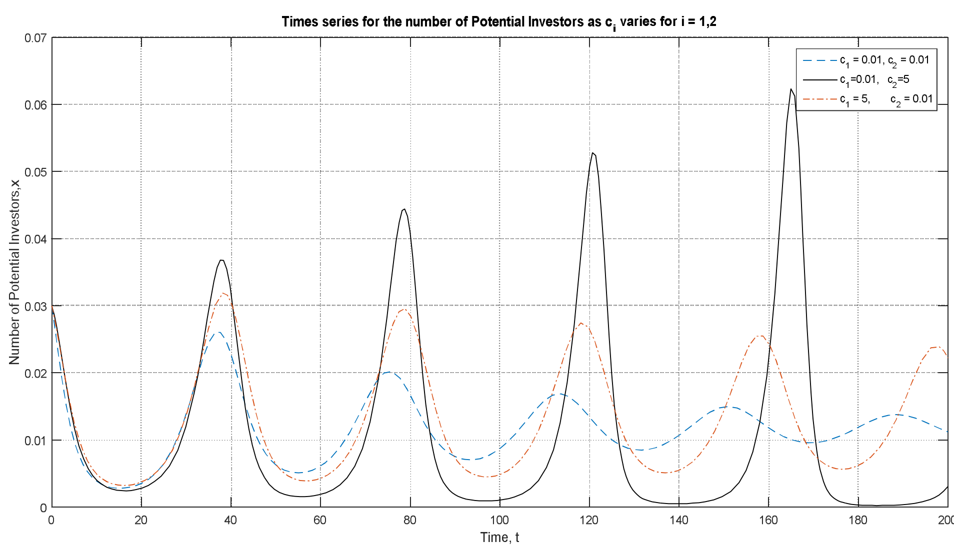

Fig. 5. Time series for the number of potential investors in datasets (1), (2) and (3) as the investor persuasion parameter, c_i varies for $i = 1, 2$. Interesting behaviour occurs particularly in dataset (2) where $c_1 = 0.01, c_2 = 5$. The potential investor population experiences the largest, increasing fluctuations when the honest expert persuasion rate is much lower than that for dishonest experts. The infection of fraud in experts influences increases the number of potential investors in the system

4 Conclusion

This model with investor interactions in the presence of fraud, can provide valuable information to safe-guard the economy from financial crises. Social networks among investors and financial experts are an integral part of the complex dynamics of these systems. Therefore, stability analyses attempt to provide investor guidance in the presence of financial predators. Parameter regions which lie in unstable regions present a mechanism to avoid incurred financial losses.

This model is the start of a bigger scheme of financial dynamical analysis. It can be improved by the addition of other financial parameters, as well as stochastic effects. This is the direction for future work. Different functional responses can also be applied in a spatial sense similar to Costner et al. [30], where prey densities in different regions have an effect on predatory activity. Since finanical eco-systems are complex, dynamic envirnoments, these functions may change from time to time.

In addition to this, a switching mechanism can create a more realistic model. Mehlum et al. [26] explained that entrepreneurs are less likely to choose predatory activities depending on the probability of businesses being the most profitable choices in the long - run. This idea can support the idea that financial experts may switch between the states of honesty or dishonesty, depending on profitability of the investment.

Hence, this work can be used to partially model and explain the dynamics in real-life fraudulent investment scenarios. Policy makers can test how different cases of parameters would affect the long – term populations of investors and financial experts in the model. The value of stability and time series analysis provide an interesting detection system to determine and predict the presence any fraudulent investment activities.

Competing Interests

Authors have declared that no competing interests exist.

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