



# Effective Self-interaction of Dark Matter from Gravitational Scattering

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## Abstract

I show that gravitational scattering of dark matter objects of mass  $\sim 10^4 M_\odot$  and speeds of  $\sim 10 \text{ km s}^{-1}$ , provides the cross section per unit mass required in self-interacting dark matter models that alleviate the small-scale structure challenges to the collisionless cold dark matter model. For primordial objects of mass  $10^4 M_4 M_\odot$ , moving at the velocity dispersion characteristic of dwarf galaxies,  $10 v_1 \text{ km s}^{-1}$ , the cross section per unit mass for gravitational scattering is  $\sim 10 (M_4/v_1^4) \text{ cm}^2 \text{ g}^{-1}$ . The steep decline in interaction with increasing velocity explains why self-interaction is not evident in data on massive galaxies and clusters of galaxies.

*Unified Astronomy Thesaurus concepts:* Cold dark matter (265)

## 1. Introduction

Self-interacting dark matter (SIDM; Spergel & Steinhardt 2000; Firmani et al. 2000) could solve the small-scale structure challenges to the standard cosmological model of cold dark matter (Kaplinghat et al. 2016; Bullock & Boylan-Kolchin 2017). Many studies over the past two decades demonstrated that a self-interaction cross section per unit mass,  $\sigma/m$ , in the range  $(1-10) \text{ cm}^2 \text{ g}^{-1}$  modifies the expected dark matter cusps to central cores—as suggested by observations of dwarf galaxies (Davé et al. 2001; Colín et al. 2002; Vogelsberger et al. 2012; Rocha et al. 2013; Zavala et al. 2013; Vogelsberger et al. 2014; Fry et al. 2015; Kamada et al. 2017; Creasey et al. 2017; Robles et al. 2017; Sameie et al. 2018; Tulin & Yu 2018; Fitts et al. 2019; Sameie et al. 2020; Meshkizde et al. 2022; Silverman et al. 2022), and resolves the “too-big-to-fail” challenge (Vogelsberger et al. 2012; Kaplinghat et al. 2019; Turner et al. 2021). We note, however, that the uncertainty in modeling the baryonic component is still large enough to also offer a possible solution to these small-scale challenges (Bullock & Boylan-Kolchin 2017).

Since there is no apparent discrepancy with cold dark matter on the scales of massive galaxies or groups (with  $v \gtrsim 10^2 \text{ km s}^{-1}$ ) and clusters of galaxies (with  $v \sim 10^3 \text{ km s}^{-1}$ ), Loeb & Weiner (2011) proposed a decade ago that the interaction might be mediated by a Yukawa potential, declining inversely with velocity to the fourth power,  $\propto v^{-4}$ , as expected in some dark sector extensions to the standard model of particle physics (see also Chaffey et al. 2021). Most recently, velocity-dependent cross sections with values  $\gtrsim 5 \text{ cm}^2 \text{ g}^{-1}$  at  $v \lesssim 10 \text{ km s}^{-1}$  were motivated to explain the dynamical properties of Milky Way satellites (Silverman et al. 2022), but with the provision that the interaction must drop sharply with velocity to  $\ll 1 \text{ cm}^2 \text{ g}^{-1}$  in massive systems (Kaplinghat et al. 2016; Sankar Ray et al. 2022).

Here we point out that the normalization and velocity dependence of the cross section per unit mass required to alleviate the small-scale structure challenges to the cold dark matter model is provided naturally by gravitational scattering if the dark matter is composed of objects in the mass range of

$10^3-10^4 M_\odot$  and a physical size  $\lesssim 1 \text{ pc}$ . The considerations are presented in the next section and the implications are summarized in the concluding section.

## 2. Cross Section for Gravitational Scattering

The gravitational cross section for scattering of compact objects with mass  $m$  and characteristic velocity  $v$  is given by (Spitzer 1962; Binney & Tremaine 1987)

$$\sigma = 8\pi \times \left( \frac{Gm}{v^2} \right)^2 \ln \Lambda, \quad (1)$$

where  $\ln \Lambda = \ln(b_{\text{max}}/b_{\text{min}})$  is the Coulomb logarithm, determined by the ratio between the maximum and minimum values of the impact parameter, with  $b_{\text{max}} \sim (4\pi\rho/3m)^{-1/3}$  being the average separation between objects at a mass density  $\rho$ , and  $b_{\text{min}} \sim 2Gm/v^2$  is the impact parameter for a  $90^\circ$  deflection. The characteristic parameters in the cores of dwarf galaxies,  $v \sim 10 \text{ km s}^{-1}$  and  $\rho \sim 3 \times 10^7 M_\odot \text{ kpc}^{-3}$ , yield  $\ln \Lambda \sim 4$  for  $m \sim 10^4 M_\odot$ .

Dividing the cross section by the object’s mass, we get

$$\frac{\sigma}{m} = 10 \text{ cm}^2 \text{ g}^{-1} \left[ \frac{(m/10^4 M_\odot)}{(v/10 \text{ km s}^{-1})^4} \right]. \quad (2)$$

Remarkably, for the mass range of  $m \sim (10^3-10^4) M_\odot$ , gravitational scattering provides the normalization and velocity dependence required for alleviating the small-scale structure challenges to the cold dark matter model.

The physical size of the dark matter object,  $R$ , must be smaller than the minimum impact parameter for their gravitational scattering

$$R_{\text{max}} = \left( \frac{2Gm}{v^2} \right) = 1 \text{ pc} \left[ \frac{(m/10^4 M_\odot)}{(v/10 \text{ km s}^{-1})^2} \right], \quad (3)$$



implying a mass density inside each object that exceeds the value

$$\begin{aligned} \rho_{\min} &= \left[ \frac{m}{(4\pi/3)R_{\max}^3} \right] \\ &= 2 \times 10^{12} M_{\odot} \text{ kpc}^{-3} \left[ \frac{(v/10 \text{ km s}^{-1})^6}{(m/10^4 M_{\odot})^2} \right]. \end{aligned} \quad (4)$$

This minimum density of a characteristic value,  $\sim 2 \times 10^{-19} \text{ g cm}^{-3}$ , is  $7 \times 10^{10}$  larger than the mean cosmic density of matter in the present-day universe and corresponds to a minimum formation redshift of  $\gtrsim 700$ , around the cosmic epoch of hydrogen recombination. The origin of such dark matter objects must therefore be primordial since the standard power spectrum of density fluctuations forms the first virialized minihalos at redshifts  $z \sim 70$  (Loeb 2010; Loeb & Furlanetto 2013; Loeb 2014).

### 3. Implications

We have shown that gravitational scattering of compact objects could provide the cross section per unit mass required in self-interacting dark matter models that alleviate the small-scale structure challenges to the collisionless cold dark matter model. For primordial objects of mass  $10^3\text{--}10^4 M_{\odot}$ , the cross section for gravitational scattering is  $\sim 1\text{--}10 \text{ cm}^2 \text{ g}^{-1}$  at the velocity dispersion characteristic of dwarf galaxies,  $\sim 10 \text{ km s}^{-1}$ . The sharp decline in the cross section at higher velocities,  $\propto v^{-4}$ , explains why self-interaction is not evident in data on massive galaxies or clusters of galaxies (Kaplinghat et al. 2016). Much larger values of the cross section, corresponding to a higher mass  $m$ , are disfavored since they trigger gravothermal core collapse (Turner et al. 2021).

Ultra-faint galaxies, such as Segue 1 and 2 (Walker et al. 2009), possess velocity dispersions of a few  $\text{km s}^{-1}$  where scattering should be more pronounced. They offer excellent laboratories for testing the model proposed here. Additional constraints on the existence of massive dark matter objects can be derived from the comparison between data (from Gaia, HST, JWST, and LSST) and numerical simulations of cold streams in a clumpy Milky Way halo (Bonaca et al. 2020, 2021; Banik et al. 2021; Banik & Bovy 2021).

Primordial black holes (PBHs) in the required mass range are constrained by microlensing of supernovae and of stars, as well as by wide binaries and X-ray binaries; for a compilation of all related limits, see Figure 1 in Carr & Kuhnel (2021). The characteristic value of  $R_{\max}$  in Equation (3) is larger than the Einstein radius of microlenses or the typical separation of wide binaries; in addition, extended objects need not trigger substantial X-ray luminosity from accretion of baryons. Therefore, the above PBH constraints might be relaxed for objects that are not as compact as black holes. In order to resolve the small-scale structure challenges, the objects under consideration here must make most of the dark matter.

If the required objects resulted from a cosmological phase transition at a temperature  $T$ , then their mass is expected to reflect the horizon mass

$$m_{\text{H}} \sim 10^4 M_{\odot} \left( \frac{T}{2 \text{ MeV}} \right)^{-2}. \quad (5)$$

Interestingly, the required mass range is naturally realized during the weak-interaction epoch, after the quantum chromo

dynamics (QCD) phase transition at  $T \sim 200 \text{ MeV}$  and before neutrino decoupling at  $\sim 1 \text{ MeV}$ . This is well above the minimum redshift for the production of cold dark matter (Sarkar et al. 2015).

Another possible origin of the dark matter clumps is that they were the first objects to collapse gravitationally after cosmological recombination (in the redshift range of  $z \sim 10^2\text{--}10^3$ ) as a result of large density fluctuations on their mass scale. In this case, the dark matter can still be collisionless at the elementary particle level. These objects would evade microlensing and wide-binary constraints on PBHs because they are extended and fluffy. The standard cosmological model makes the first collapsed objects at redshift  $z \sim 70$  (Barkana & Loeb 2001; Naoz & Barkana 2007; Loeb & Furlanetto 2013; Loeb 2016). Interestingly, the baryonic Jeans mass is  $\sim 10^4 M_{\odot}$  at that redshift (Loeb 2016), but larger than standard primordial fluctuations are needed to clump most of the dark matter into the objects of subparsec size discussed here.

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