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# Investigating the collective impact of postponement, scrap, and external suppliers on multiproduct replenishing decision

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ABSTRACT

This study examines the collective impact of postponement, scrap, and subcontracting standard components on the multiproduct replenishing decisions. Rapid response, desirable quality, and various goods guide the client's demands in today's competitive market. Therefore, many manufacturing firms search for alternative fabrication and outsourcing strategies during the production planning stage to satisfy the client's expectations, minimize fabrication-inventory costs, and smoothen machine utilization. To effectively help producers meet today's client's needs and enhance their competitive advantage, we develop a two-stage multiproduct replenishing system incorporating scraps, standard parts subcontracting, commonality, and delayed differentiation. To reduce the production uptime, stage one has a hybrid fabrication process for the common components (i.e., a partial outsourcing strategy), and stage two manufactures the finished multiproduct. In-house fabrication processes in both stages are imperfect; a screening process detects and removes scraps to maintain the finished batch quality. We determine the cost-minimized operating cycle. The findings reveal the collective impact of postponement, scrap, and external suppliers on this multi-product replenishment problem and can be used to facilitate production planning and decision-making.

## Notation

$T$ = cycle length,
$E[T]$ = the expected cycle length,
$TC(T)$ = total system cost per cycle,
$E[TC(T)]$= the expected total system per cycle,
$E[TCU(T)]$ = the expected annual system cost.

Notation used in stage-1 concerning the standard parts:

$\lambda_0$ = annual demand,
$Q_0$ = in-house lot size,
$S_0$ = setup time,
$t_0^*$ = optimal uptime,
$t_{1,0}$ = in-house uptime,
$t_{2,0}$ = stock depleting time,
$H_{1,0}$ = level of stock when uptime completes,
$h_{1,0}$ = unit holding cost,

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$H_{3,0}$ = level of stock when outsourced items received,
$h_{4,0}$ = unit safety stock's holding cost,
$i_0$ = relating ratio for holding cost.

Notation used in stage-2 concerning end product $i$ (for $i = 1, 2, \ldots, L$):

$Q_i$ = lot-size,
$C_i$ = unit cost,
$S_i$ = setup time,
$K_i$ = setup cost,
$t_i^*$ = summation of optimal uptimes,
$t_{1,i}$ = uptime,
$h_{1,i}$ = unit holding cost,
$t_{2,i}$ = stock depleting time,
$h_{4,i}$ = safety stock's unit holding cost,
$H_{1,i}$ = stock level when uptime ends,
$H_i$ = standard parts' level each uptime ends,
$I(t)_i$ = stock level at time $t$.

## 1. Introduction

In today's turbulent markets, clients need a variety of goods of timely and desired quality. Hence, to satisfy clients' needs and keep the overall system expenses minimal, the manufacturing firms continuously seek the most appropriate production scheme. When various products have a commonality feature, the delayed differentiation strategy is often evaluated to make all needed standard, intermediate components for these end products and fabricate each specific finished product in the second stage. Swaminathan (2001) studied the modularization of a firm's products and their relevant fabrication processes. The impact of four dimensions, namely: part, product, procurement, and process, on the degree of customization was explored to reveal their benefits in terms of rapid response and cost minimization. Davila and Wouters (2007) studied the potential advantages of implementing the postponement scheme. The authors used regression analysis on actual data from an electronic component producer that adopted a postponement scheme in its supply chain to study the potential benefits. They found that a higher degree of postponement implementation resulted in a better performance of cost savings and on-time deliveries. Ngniatedema et al. (2015) built a delayed customization model to explore the raw materials supplier's lead time performance for coping with the uncertainties in demands. The authors looked into the cost-effective delayed differentiation approach under the constraint of the raw materials delivery window. They revealed the significant impact of delivery performance on determining the optimal postponement point. They further indicated the correlations between the level of customer services in different fabrication stages and the decision to delay the customization point. They used a real-world case to illustrate their proposed framework application. Le Pape and Wang (2020) investigated the influence of product differentiation on principal shareholders' conflicts. The authors exhibited the hierarchy of battles among principal shareholders and explored the postponement strategy's price and quantity competition effect. Other works (Tookanlou and Wong, 2020; Chiu et al., 2020; Bolaños and Barbalho, 2021; Ghasemy Yaghin and Goh, 2021; Malladi et al., 2021; Ramón-Lumbierres et al., 2021; Chiu et al., 2022a) considered the effect of postponement strategy on production operations, planning, and controlling of multiproduct fabricating systems.

Since fabricating required standard components needs longer uptime, implementing external sources to provide a portion of common components' batch (i.e., applying an outsourcing strategy) may reduce uptime. Abraham and Taylor (1996) examined the influence of wage, potential benefit, demand instability, skills, and capability of an external contractor on the outsourcing decision. The authors used empirical analyses to explore the relationship between the service contracting out choice and a firm's internal labor capacity. Nembhard et al. (2003) designed an actual options model that formulated and assessed the financial benefits of product outsourcing. The authors used the Monte Carlo technique to simulate a problem with three state variables, and the resulting market dynamic view can facilitate proper outsourcing choices. Using a real case, the authors demonstrate how their model works and give a better long-term look at outsourcing. Çınar and Güllü (2012) studied an uncertain-capacity production-inventory system incorporating the availability of advance capacity (outsourcing option) to hedge against unstable demands. The authors planned the production using the regular capacity and then enhanced their plan with an outsourcing option to derive an order-up-to optimal level policy. Numerical demonstrations exhibited the applicability and benefits of using their model. Other studies (Dekker et al., 2020; Gupta and Ivanov, 2020; Iqbal et al., 2020; Mabrouk, 2020; Chiu, et al., 2021a,b; Chiu et al., 2022b; Singagerda et al., 2022) examined the effect of different outsourcing options on supply chains, business operations, and manufacturing systems.

To ensure the finished batch's quality, many producers screen their fabrication outputs to identify and remove the evitable faulty items. Nguyen and Murthy (1989) built a repair-replace decision model for the standard warranty policy. They aimed to decide the best way to fulfill the client's product warranty. Geren and Lo (1998) addressed the automated printed circuit board assembly (PCBA) rework cell problem. Through the choice of rework tool and techniques and interface of the control equipment, the authors exhibited the approach for building a robotic PCBA rework cell. Chelbi and Rezg (2006) investigated a flawed fabrication-inventory system conditional on a random breakdown and a minimum needed availability "A" level. The product unit takes preventive maintenance at a fixed time interval "T" or a correction action at breakdown occurrence. A buffer inventory level "h" is accumulated to avoid the stock-out situation in either case. Accordingly, the authors determined the cost-minimized policy in terms of "T" and "h" under the constraint of "A." Aslani et al. (2017) examined an Economic Order Quantity (EOQ) inventory model featuring partial back order and a random yield at the supplier side. The authors proposed two strategies at a cost to improve the mean and variance of the supplier's defective rate. Numerical illustrations supported their proposals and solution methods. Extra works (Abdelall et al., 2020; Dewi et al., 2021; Di Nardo et al., 2021; Okfalisa et al., 2021; Perarasi et al., 2021; Suroso et al., 2021; Allaham and Dalalah, 2022; Lin et al., 2022) addressed the influence of various imperfection in fabrication processes on the manufacturing systems and production planning and control. Inspired by the urgent need to assist today's manufacturing firms in making cost-effective and efficient multi-item production decisions, this study develops a model to serve this purpose. We aim to help producers meet the recent trends in clients' requirements concerning rapid response and various merchandise. Few studies mentioned above focused on this specific area; this study aims to bridge the research gap.

## 2. Materials and methods

This study investigates the combined effect of postponement, scrap, and external providers for common components and scraps on the multiproduct replenishing decision. The description, assumption, mathematical modeling, and analysis are explicitly provided in the following subsections:

### 2.1. Assumption and model description

Suppose a common intermediate component occurs in planning multiproduct fabrication. In that case, the management often searches for an alternative producing scheme/process redesign, such as the postponement strategy to delay differentiation seeking cost benefits, and/or reducing cycle length/response time. This study investigates a two-stage multiproduct batch production problem incorporating commonality, postponement, common parts' external suppliers, and scrap into the production processes. Assume a two-stage fabricating process produces $L$ end items to satisfy demands $\lambda_i$ (where $i = 1, 2, \ldots, L$). Stage one makes all end products' needed common parts at a rate $P_{1,0}$ per year. Stage two fabricates $L$ specific end items at a separate $P_{1,i}$ per year. We assume a constant of the standard component's completion rate $\gamma$ (compared with its finished good.) For example, if $\gamma = 50\%$ and because $P_{1,i}$ and $P_{1,0}$ depend on $\gamma$, $P_{1,i}$ and $P_{1,0}$ become two times their ordinary fabricating rate in a single-stage system. To reduce stage one's long uptime, we contract out a $\pi_0$ percentage of the necessary standard parts per cycle. Associating with this subcontracting policy, we have a different fixed setup expense $K_{\pi0}$ and unit expense $C_{\pi0}$ (Eqs. (1) and (2)).

$$K_{\pi0} = K_0\left(1+\beta_{1,0}\right) \tag{1}$$

$$C_{\pi0} = \left(1+\beta_{2,0}\right)C_0 \tag{2}$$

where $C_0$, $\beta_{1,0}$, $K_0$, and $\beta_{2,0}$ represent the setup and unit costs of the in-house common parts production and the linking factors for these cost-relevant parameters. For instance, $\beta_{2,0} = 0.16$ represents that $C_{\pi0}$ is 16% higher than the $C_0$, and $\beta_{1,0} = -0.4$ means that $K_{\pi0}$ is 40% less than $K_0$, etc. We also assume the schedule receipt of subcontracting components is at stage one uptime ending time (see Fig. 1). Further assumption includes the existence of random faulty percentages $x_0$ and $x_i$ in each stage. So, the faulty items' fabricating rates in stage-two and –one are $d_{1,i}$ and $d_{1,0}$ ($d_{1,i} = P_{1,i}x_i$ and $d_{1,0} = P_{1,0}x_0$). The unit disposal costs for defect items are $C_{S,0}$, and $C_{S,i}$.

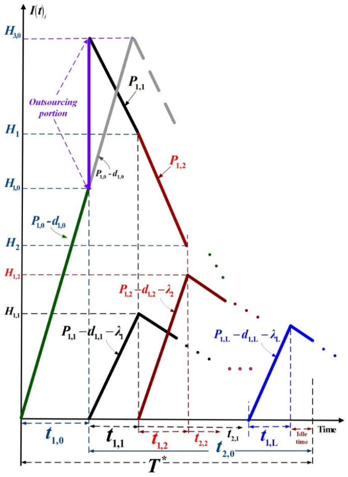

**Fig. 1.** Stock status of our proposed two-stage multiproduct batch production problem incorporates commonality, postponement, scrap, and external supplier compared with the same problem with no outsourcing option (in grey)

Fig. 1 displays our model's stock level. It exposed that stage one's stock-level reaches $H_{1,0}$ when uptime ends. It rises to $H_{3,0}$ upon receipt of outsourced stocks. For $i = 1, 2, \ldots, L$, stage two's stock status reaches $H_{1,i}$ when its uptime ends. It begins to decrease during $t_{2,i}$. Without permitting shortages, we must have $P_{1,i} - d_{1,i} - \lambda_i > 0$ and $P_{1,0} - d_{1,0} > 0$. Fig. 2 exhibited the level of faulty stocks. It shows the maximal faulty items' level is $(d_{1,0}\, t_{1,0})$ and maximal end product $i$'s level is $(d_{1,i}\, t_{1,i})$.

### *2.2. Modeling, formulations, and analysis*

The standard parts are consumed to satisfy the requirements of manufacturing $L$ end products (see their stock status in Fig. 3 and refer to Fig. 1 on its status in $t_{2,0}$).

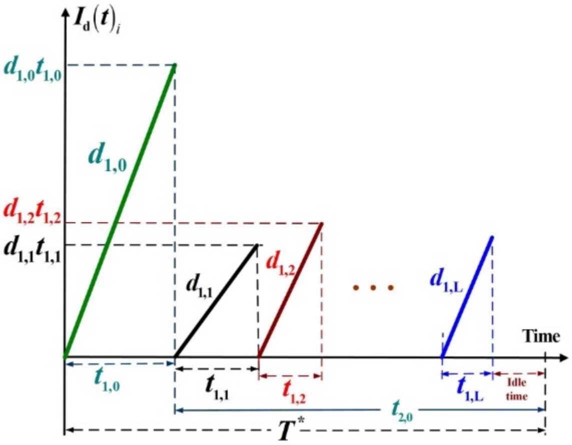

**Fig. 2.** The defective stocks' status

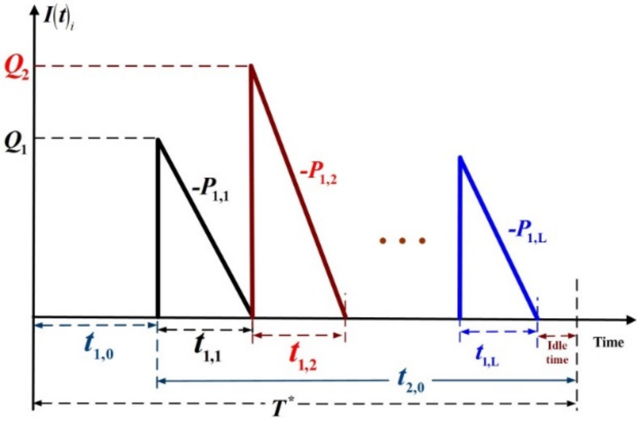

**Fig. 3.** Common part's status as required from fabricating end products in stage 2

From stage two's assumption, one can obtain formulas (3) to (7) (for $i = 1, 2, \ldots, L$).

$$Q_i = \frac{\lambda_i T}{1 - E[x_i]} \tag{3}$$

$$H_{1,i} = t_{1,i}\left(P_{1,i} - d_{1,i} - \lambda_i\right) \tag{4}$$

$$T = t_{1,i} + t_{2,i} \tag{5}$$

$$t_{1,i} = \frac{Q_i}{P_{1,i}} = \frac{H_{1,i}}{P_{1,i} - d_{1,i} - \lambda_i} \tag{6}$$

$$t_{2,i} = \frac{H_{1,i}}{\lambda_i} \tag{7}$$

According to Eq. (3), total needed common parts is shown in Eq. (8).

$$H_{3,0} = \sum_{i=1}^{L} \frac{\lambda_i T}{1 - E[x_i]} = \sum_{i=1}^{L} Q_i \tag{8}$$

From stage-1's assumption and Figures 1 to 3, one also finds formulas (13) to (18).

$$\lambda_0 = \frac{\sum_{i=1}^{L} Q_i}{T} \tag{9}$$

$$H_{3,0} = H_{1,0} + \pi_0 \left( \sum_{i=1}^{L} Q_i \right) \tag{10}$$

$$Q_0 = \frac{H_{1,0}}{1 - E[x_0]} \tag{11}$$

$$H_{1,0}=\left(\sum_{i=1}^{L}Q_i\right)(1-\pi_0) \tag{12}$$

$$H_{1,0}=t_{1,0}\left(P_{1,0}-d_{1,0}\right) \tag{13}$$

$$t_{1,0}=\frac{H_{1,0}}{P_{1,0}-d_{1,0}}=\frac{Q_0}{P_{1,0}} \tag{14}$$

$$T=t_{1,0}+t_{2,0} \tag{15}$$

$$H_1=H_{3,0}-Q_1 \tag{16}$$

$$H_i=H_{(i-1)}-Q_i\ \left(for\ i=2,\ 3,\ ...,L\right) \tag{17}$$

$$H_L=H_{(L-1)}-Q_L=0 \tag{18}$$

## 3. Results, illustration, and discussion

The *TC*(*T*) includes the expenses in (1) stage one: the outsourcing variable and fixed costs, the in-house manufacturing variable, setup, disposal, and holding costs; (2) stage two: the summation of fabricating setup, variable, holding, and disposal costs for *L* end products:

$$\begin{aligned}TC(T)=&K_{\pi0}+C_{\pi0}\pi_0\left(\sum_{i=1}^{L}Q_i\right)+K_0+C_0Q_0+h_{4,0}x_0Q_0T+C_{S,0}x_0Q_0\\&+h_{1,0}\left[\sum_{i=1}^{L}\left[\frac{Q_i}{2}\left(t_{1,i}\right)+H_i\left(t_{1,i}\right)\right]+\frac{t_{1,0}d_{1,0}}{2}\left(t_{1,0}\right)+\frac{t_{1,0}H_{1,0}}{2}\right]\\&+\sum_{i=1}^{L}\Bigg\{K_i+C_iQ_i+h_{4,i}x_iQ_iT+C_{S,i}\left(Q_ix_i\right)+h_{1,i}\left[\frac{H_{1,i}}{2}\left(t_{2,i}\right)+\frac{d_{1,i}t_{1,i}}{2}\left(t_{1,i}\right)+\frac{H_{1,i}t_{1,i}}{2}\right]\Bigg]\Bigg\}\end{aligned} \tag{19}$$

Substitute Eqs. (3) to (18) in Eq. (19) and apply $E[x_0]$ to $E[TCU(T)]$ (i.e., $E[TC(T)]/E[T]$), and the following $E[TCU(T)]$ is gained with additional derivations:

$$\begin{aligned}E\left[TCU(T)\right]=&\left\{\begin{array}{l}\frac{K_{\pi0}}{T}+\frac{K_0}{T}+C_0E_{00}\left(1-\pi_0\right)\lambda_0+C_{S,0}E_{10}\left(1-\pi_0\right)\lambda_0+h_{4,0}E_{10}\lambda_0T\left(1-\pi_0\right)+h_{1,0}\sum_{i=1}^{L}\left[\frac{E_{0i}{}^2\lambda_i{}^2T}{2P_{1,i}}\right]\\+h_{1,0}\left(\frac{1}{P_{1,0}}\right)\frac{\left(E_{00}\right)^2\lambda_0{}^2\left(1-\pi_0\right)^2T}{2}+C_{\pi0}\left(\pi_0\lambda_0\right)+h_{1,0}\sum_{i=1}^{L}\Bigg[\left(\frac{\lambda_iT}{P_{1,i}}E_{0i}\right)\left(\sum_{i=1}^{L}\left(\lambda_iE_{0i}\right)-\sum_{j=1}^{i}\left(\lambda_jE_{0j}\right)\right)\Bigg]\Bigg]\end{array}\right\}\\&+\sum_{i=1}^{L}\left\{\frac{K_i}{T}+h_{4,i}T\lambda_iE_{1i}+C_i\lambda_iE_{0i}+C_{S,i}\lambda_iE_{1i}+h_{1,i}\left[1-\frac{\lambda_i\left(1-2E\left[x_i\right]\right)E_{0i}{}^2}{P_{1,i}}\right]\frac{T\lambda_i}{2}\right\}\end{aligned} \tag{20}$$

where $E_{00}$, $E_{10}$, $E_{0i}$, $E_{1i}$, and $E_{0j}$ stand for the following:

$$E_{0i}=\frac{1}{\left(1-E\left[x_i\right]\right)}\text{ and }E_{1i}=\frac{E\left[x_i\right]}{\left(1-E\left[x_i\right]\right)}\ \left(for\ \ i=1,...,L\right);E_{00}=\frac{1}{\left(1-E\left[x_0\right]\right)};$$

$$E_{0j}=\frac{1}{\left(1-E\left[x_j\right]\right)}\ \left(for\ \ j=1,...,i\right);E_{10}=\frac{E\left[x_0\right]}{\left(1-E\left[x_0\right]\right)}.$$

### *3.1. Solution procedure*

By computing the 1st and 2nd derivatives of $E[TCU(T)]$, one gains the following Eqs. (21) and (22):

$$\begin{aligned}\frac{dE\left[TCU(T)\right]}{dT}=&\left\{\begin{array}{l}-\frac{K_{\pi0}}{T^2}-\frac{K_0}{T^2}+h_{1,0}\frac{\left(E_{00}\right)^2\left(1-\pi_0\right)^2\lambda_0{}^2}{2P_{1,0}}+h_{4,0}E_{10}\lambda_0\left(1-\pi_0\right)\\+h_{1,0}\sum_{i=1}^{L}\left[\frac{\lambda_i{}^2E_{0i}{}^2}{2P_{1,i}}\right]+h_{1,0}\sum_{i=1}^{L}\Bigg[\left(\frac{E_{0i}\lambda_i}{P_{1,i}}\right)\left(\sum_{i=1}^{L}\left(E_{0i}\lambda_i\right)-\sum_{j=1}^{i}\left(E_{0j}\lambda_j\right)\right)\Bigg]\Bigg]\end{array}\right\}\\&+\sum_{i=1}^{L}\left\{\frac{h_{1,i}\lambda_i}{2}\left[1-\frac{\lambda_iE_{0i}{}^2\left(1-2E\left[x_i\right]\right)}{P_{1,i}}\right]+h_{4,i}\lambda_iE_{1i}-\frac{K_i}{T^2}\right\}\end{aligned} \tag{21}$$

$$\frac{d^2E\left[TCU\left(T\right)\right]}{dT^2}=\frac{2K_{\pi 0}}{T^3}+\sum_{i=1}^{L}\left\{\frac{2K_i}{T^3}\right\}+\frac{2K_0}{T^3}>0 \tag{22}$$

Since $K_0$, $K_{\pi 0}$, and $K_i$ are positive and $T$ is positive, $E[TCU(T)]$ is convex. Now, by setting Eq. (21) = 0, one can derive the following $T^*$:

$$T^*=\sqrt{\frac{\left(\sum_{i=1}^{L}K_i\right)+\left(\beta_{1,0}+2\right)K_0}{\begin{array}{l}h_{1,0}\sum_{i=1}^{L}\left[\left(\frac{E_{0i}\lambda_i}{P_{1,i}}\right)\left(\sum_{i=1}^{L}\left(E_{0i}\lambda_i\right)-\sum_{j=1}^{i}\left(E_{0j}\lambda_j\right)\right)\right]+h_{4,0}\left(1-\pi_0\right)\lambda_0E_{10}+h_{1,0}\frac{\lambda_0^{\,2}\left(E_{00}\right)^2\left(1-\pi_0\right)^2}{2P_{1,0}}\\ +h_{1,0}\sum_{i=1}^{L}\left[\frac{\lambda_i^{\,2}E_{0i}^{\,2}}{2P_{1,i}}\right]+\sum_{i=1}^{L}\left\{h_{1,i}\frac{\lambda_i}{2}\left[1-\frac{\left(1-2E\left[x_i\right]\right)\lambda_iE_{0i}^{\,2}}{P_{1,i}}\right]+h_{4,i}\lambda_iE_{1i}\right\}\end{array}}} \tag{23}$$

### *3.2. The illustration*

This section offers a numerical example of how postponement, scrap, and external supplies of standard parts affect the five-product replenishment decision. The assumed values of system parameters in the first and second manufacturing stages are exhibited in Tables 1 and 2, respectively. For comparison purposes, Table B-1 (in Appendix B) shows the corresponding parameters' values for a problem using the single-stage production scheme.

**Table 1**

Parameters' values in the first manufacturing stage

| $P_{1,0}$ | $C_0$ | $h_{1,0}$ | $\lambda_0$ | $C_{S,0}$ | $h_{4,0}$ | $\gamma$ |
|---|---|---|---|---|---|---|
| 120000 | \$40 | \$8 | 17406 | \$10 | \$8 | 0.5 |
| $\pi_0$ | $K_0$ | $\beta_{1,0}$ | $x_0$ | $i_0$ | $\beta_{2,0}$ | $\delta$ |
| 0.4 | \$8500 | -0.7 | 2.5% | 0.2 | 0.4 | 0.5 |

**Table 2**

The assumed parameters' values in the second manufacturing stage

| Product $i$ | $x_i$ | $P_{1,i}$ | $h_{1,i}$ | $K_i$ | $i_i$ | $C_i$ | $C_{S,i}$ | $\lambda_i$ | $h_{4,i}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.5% | 112258 | \$16 | \$8500 | 0.2 | \$40 | \$10 | 3000 | \$16 |
| 2 | 7.5% | 116066 | \$18 | \$9000 | 0.2 | \$50 | \$15 | 3200 | \$18 |
| 3 | 12.5% | 120000 | \$20 | \$9500 | 0.2 | \$60 | \$20 | 3400 | \$20 |
| 4 | 17.5% | 124068 | \$22 | \$10000 | 0.2 | \$70 | \$25 | 3600 | \$22 |
| 5 | 22.5% | 128276 | \$24 | \$10500 | 0.2 | \$80 | \$30 | 3800 | \$24 |

To search for the optimal $T^*$ and $E[TCU(T^*)]$ for our proposed multiproduct replenishing model and demonstrate how it works, one can apply the formulas (23) and (20) to obtain $T^*$ = 0.5278 years and $E[TCU(T^*)]$ = \$2,228,012. Fig. 4 disclosed the behavior of $E[TCU(T)]$ relating to $T$. It shows that as $T$ departs from $T^*$ in both ways, $E[TCU(T)]$ increases significantly.

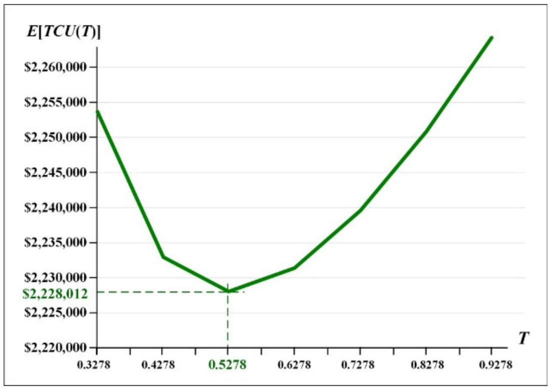

**Fig. 4.** Behavior of $E[TCU(T)]$ relating to $T$

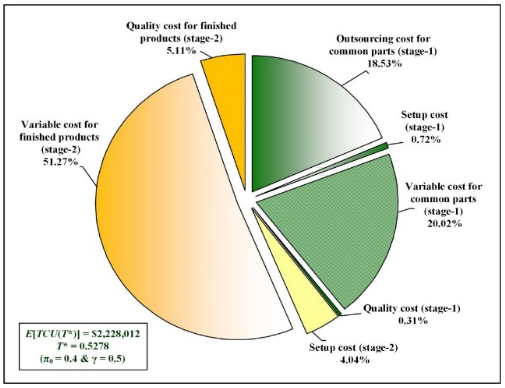

**Fig. 5.** Breakup of system cost $[TCU(T^*)]$

The contributors to system costs are investigated separately, and the breakup outcomes are illustrated in Fig. 5. It indicates that major donors are the variable cost for finished products (51.27%) and the standard components' variable cost (20.02%), and its outsourcing cost (18.53%). These major ones add up to 89.82%. Due to random defective items in both stages, the quality cost contributes a total 5.42% to $E[TCU(T^*)]$ (i.e., 5.11% (in stage-2) plus 0.31% (in stage-1)).

### 3.2.1. *The effect of outsourcing strategy on the studied problem*

Since an outsourcing strategy is implemented in this study, its direct effect on the uptime $t_0$* of stage one is explored and illustrated in Fig. 6. It shows that for $\pi_0 = 0.4$, $t_0$* decreases from 0.0779 to 0.0487 (year), it is a decline of 37.48% (see column (B) of Table C-1); it also indicates that $t_0$* knowingly drops as $\pi_0$ increases.

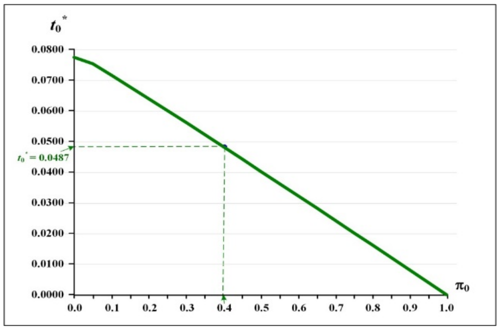

**Fig. 6.** The influence of variations in $\pi_0$ on stage one's uptime $t_0$*

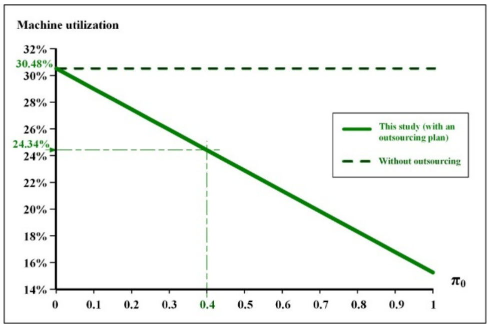

**Fig. 7.** The analytical results of utilization vis-à-vis $\pi_0$

Fig. 7 reveals the analytical results of the proposed system's utilization vis-à-vis $\pi_0$. It exposes that the utilization declined from 30.48% to 24.34%, a decrease of 20.14%, owing to an outsourcing strategy. It also specifies that as $\pi_0$ rises, the machine utilization decreases considerably. Fig. 8 shows the utilization for each end item relating to $\pi_0$. It demonstrates that this study can explore in-depth performance in stage two of our system. Fig. 8 discloses that each end product's utilization varied, and the differences in $\pi_0$ have no impact on these utilizations.

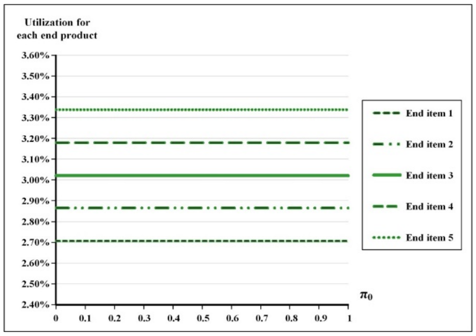

**Fig. 8.** The utilization for each end item relating to $\pi_0$

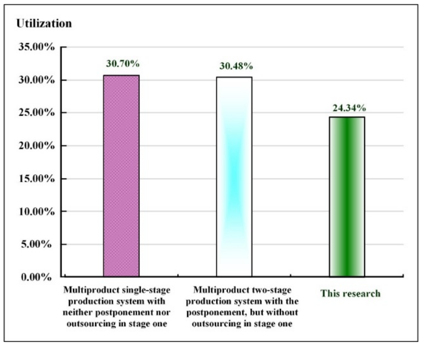

**Fig. 9.** Comparing the utilization of this research with different models

Fig. 9 compares the utilization of this research to other similar models. Because of the implementation of an outsourcing strategy, this research's utilization drops to 24.34% (as stated in Fig. 7), which is 20.14% less than that in the same two-stage system without an outsourcing plan (see Fig. 9). Moreover, it shows a decrease of 20.72% in utilization (i.e., dropping from 30.70% to 24.34%; see Fig. 9) compared to a pure single-stage system with neither postponement nor outsourcing implemented. Fig. 10 compares $E[TCU(T^*)]$ with different models. It indicated that we pay the price of a 5.33% rise in $E[TCU(T^*)]$ (i.e., growing to \$2,228,012 from \$2,115,234; see Figure 10) for a 20.14% reduction in utilization compared to a two-stage system without involving external sources (see Fig. 10). Furthermore, for a drop of 20.72% in utilization, our $E[TCU(T^*)]$ rises 1.65% (i.e., increasing from \$2,191,923 to \$2,228,012) compared to a single-stage production system with neither postponement nor outsourcing in stage one.

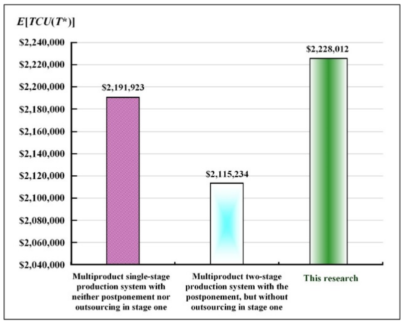

**Fig. 10.** Comparing $E[TCU(T^*)]$ of this research with different models

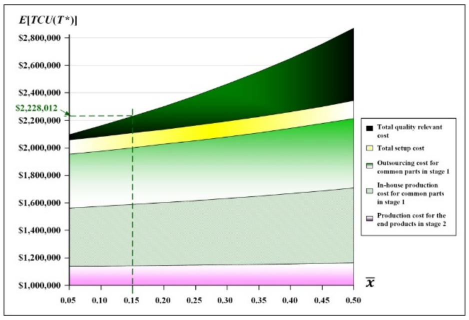

**Fig. 11.** Behavior of cost contributors to $E[TCU(T^*)]$ relating to $\overline{x}$

### 3.2.2. *The individual/collective effect of* $\bar{x}$, $\gamma$, *and* $\beta_{2,0}$

The behavior of each cost contributor of $E[TCU(T^*)]$ relating to the average in-house scrap rate $\bar{x}$ is exhibited in Fig. 11. It specifies that the total quality relevant cost upsurges significantly as $\bar{x}$ rises. The analytical capability of our model is not limited to the linear relationship between $\delta$ and $\gamma$. With the help of this model, one can investigate different relationships between $\delta$ and $\gamma$. Fig. 12 demonstrates the analytical impact of both the nonlinear and linear relationships on $E[TCU(T^*)]$. For $\gamma = 0.5$, it re-confirms the values of $E[TCU(T^*)]$ = \$2,228,012 and specifies $E[TCU(T^*)]$ = \$2,203,095 for both of the following relationship: $\delta = \gamma^1$ (linear) vs. $\delta = \gamma^{1/3}$ (nonlinear).

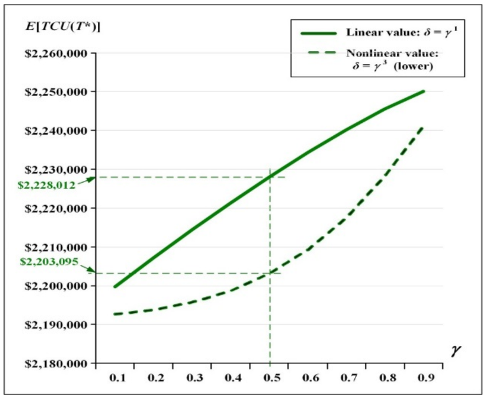

**Fig. 12.** Impact of both the nonlinear and linear relationships between $\delta$ and $\gamma$ on $E[TCU(T^*)]$

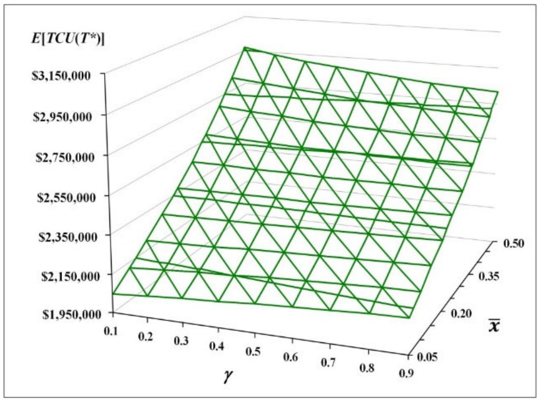

**Fig. 13.** Behavior of $E[TCU(T^*)]$ concerning the collective influence from $\bar{x}$ and $\gamma$

Fig. 13 shows the collective effect of $\gamma$ and the average in-house scrap rate $\bar{x}$ on $E[TCU(T^*)]$. It exposed that $E[TCU(T^*)]$ significantly rises as $\bar{x}$ increases. Because the add-up percentage of $\beta_{2,0} = 0.40$ when $\bar{x}$ is less than 40%, $E[TCU(T^*)]$ slightly increases as $\gamma$ rises; however, when $\bar{x}$ is large than 40%, $E[TCU(T^*)]$ marginally declines, as $\gamma$ rises. Fig. 14 shows the collective influence of the add-up percentage $\beta_{2,0}$ and the average in-house scrap rate $\bar{x}$ on $T^*$. It indicates that $T^*$ considerably declines as $\bar{x}$ rises; and $T^*$ varies insignificantly as $\beta_{2,0}$ increases.

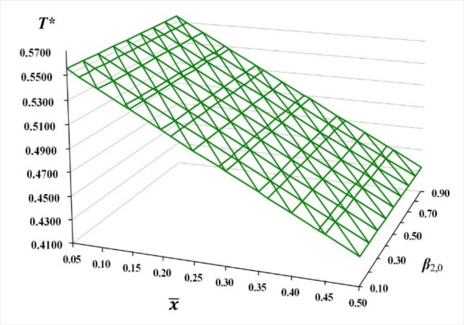

**Fig. 14.** Behavior of the optimal $T^*$ relating to the collective effect from $\bar{x}$ and $\gamma$

## 4. Conclusions

We explore the collective influence of postponement, scrap, and outsourcers for the standard components on a multiproduct replenishing system to help manufacturing firms gain a competitive advantage. We build a two-stage delayed differentiation multiproduct fabrication model featuring commonality, defective inspection, and partial outsourcing options. With the help of differential calculus, we determine the cost-minimized operating cycle (refer to Figs. 1 to 3 and the Materials and Methods section). Through the Numerical Illustration section, we reveal the various individual/collective influences of external suppliers, postponement, and scrap on this multiproduct replenishing problem. For example,

(1) the convexity of system cost, optimal replenishing cycle, and comprehensive cost contributors (see Fig. 4 and Fig. 5);

(2) the outsourcing strategy's influence on the uptime, machine utilization, and system cost of the problem (refer to Figures 6 to 10);

(3) the collective/individual effect of the completion rate $\gamma$, average scrap rate, and the add-up percentage of outsourcing item $\beta_{2,0}$ on the expected cost and optimal replenishing cycle (Fig. 11 to Fig. 14).

Incorporating a multi-end-product shipment strategy in the same context of this problem shall be a worth investigating topic for future study.

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## Appendix – A

**Table A-1**

Assumed parameters for a single-stage fabrication problem

| Product $i$ | $x_i$ | $P_{1,i}$ | $C_{S,i}$ | $\lambda_i$ | $C_i$ | $h_{1,i}$ | $i$ | $K_i$ | $h_{4,i}$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.05 | 58000 | \$20 | 3000 | \$80 | \$16 | 0.2 | \$17000 | \$16 |
| 2 | 0.10 | 59000 | \$25 | 3200 | \$90 | \$18 | 0.2 | \$17500 | \$18 |
| 3 | 0.15 | 60000 | \$30 | 3400 | \$100 | \$20 | 0.2 | \$18000 | \$20 |
| 4 | 0.20 | 61000 | \$35 | 3600 | \$110 | \$22 | 0.2 | \$18500 | \$22 |
| 5 | 0.25 | 62000 | \$40 | 3800 | \$120 | \$24 | 0.2 | \$19000 | \$24 |

## Appendix – B

**Table B-1**

Impact of differences in $\pi_0$ on system's utilization & various related parameters

| $\pi_0$ | $t_0$* (A) | (A)% drops | Utilization (B) | (B) % decline | Total uptime | $T$* | E[*TCU*($T$*)] (C) | (C) % increase | Outsourcing cost (D) | (D) / (C) % |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.0779 | - | 30.48% | - | 0.1544 | 0.5066 | \$2,115,234 | - | 0 | 0.00% |
| 0.05 | 0.0759 | -2.59% | 29.72% | -2.52% | 0.1544 | 0.5195 | \$2,133,587 | 0.87% | \$55,919 | 2.62% |
| 0.10 | 0.0721 | -7.48% | 28.95% | -5.04% | 0.1508 | 0.5208 | \$2,146,991 | 1.50% | \$106,917 | 4.98% |
| 0.15 | 0.0682 | -12.40% | 28.18% | -7.56% | 0.1471 | 0.5222 | \$2,160,424 | 2.14% | \$157,915 | 7.31% |
| 0.20 | 0.0644 | -17.35% | 27.41% | -10.09% | 0.1435 | 0.5234 | \$2,173,884 | 2.77% | \$208,914 | 9.61% |
| 0.25 | 0.0605 | -22.34% | 26.64% | -12.61% | 0.1398 | 0.5246 | \$2,187,373 | 3.41% | \$259,913 | 11.88% |
| 0.30 | 0.0566 | -27.36% | 25.87% | -15.13% | 0.1360 | 0.5257 | \$2,200,890 | 4.05% | \$310,913 | 14.13% |
| 0.35 | 0.0526 | -32.41% | 25.10% | -17.65% | 0.1322 | 0.5268 | \$2,214,436 | 4.69% | \$361,913 | 16.34% |
| **0.40** | **0.0487** | **-37.48%** | **24.34%** | **-20.14%** | **0.1284** | **0.5278** | **\$2,228,012** | **5.33%** | **\$412,915** | **18.53%** |
| 0.45 | 0.0447 | -42.60% | 23.57% | -22.69% | 0.1246 | 0.5287 | \$2,241,616 | 5.97% | \$463,916 | 20.70% |
| 0.50 | 0.0407 | -47.73% | 22.80% | -25.22% | 0.1207 | 0.5296 | \$2,255,250 | 6.62% | \$514,919 | 22.83% |
| 0.55 | 0.0367 | -52.89% | 22.03% | -27.74% | 0.1168 | 0.5304 | \$2,268,914 | 7.27% | \$565,922 | 24.94% |
| 0.60 | 0.0327 | -58.06% | 21.26% | -30.26% | 0.1129 | 0.5311 | \$2,282,607 | 7.91% | \$616,926 | 27.03% |
| 0.65 | 0.0286 | -63.26% | 20.49% | -32.78% | 0.1090 | 0.5318 | \$2,296,329 | 8.56% | \$667,930 | 29.09% |
| 0.70 | 0.0246 | -68.47% | 19.72% | -35.30% | 0.1050 | 0.5324 | \$2,310,082 | 9.21% | \$718,935 | 31.12% |
| 0.75 | 0.0205 | -73.70% | 18.95% | -37.82% | 0.1010 | 0.5329 | \$2,323,865 | 9.86% | \$769,941 | 33.13% |
| 0.80 | 0.0164 | -78.94% | 18.19% | -40.35% | 0.0970 | 0.5334 | \$2,337,677 | 10.52% | \$820,947 | 35.12% |
| 0.85 | 0.0123 | -84.20% | 17.42% | -42.87% | 0.0930 | 0.5338 | \$2,351,520 | 11.17% | \$871,954 | 37.08% |
| 0.90 | 0.0082 | -89.46% | 16.65% | -45.39% | 0.0889 | 0.5341 | \$2,365,393 | 11.83% | \$922,962 | 39.02% |
| 0.95 | 0.0041 | -94.73% | 15.88% | -47.91% | 0.0848 | 0.5343 | \$2,379,296 | 12.48% | \$973,970 | 40.94% |
| 1.00 | 0.0000 | - | 15.11% | -50.43% | 0.0747 | 0.4941 | \$2,376,702 | 12.36% | \$1,025,369 | 43.14% |