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A Novel Set of Fuzzy f-Divergence Measure-Related Intuitionistic Fuzzy Information Equalities and Inequalities

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

In the literature on fuzzy information theory, there are numerous divergence metrics and fuzzy information. Disparities are crucial for determining relationships. Here, we'll discuss some fresh information inequalities related to fuzzy measures and how they apply to the detection of patterns. With the aid of the fuzzy f-divergence measure and Jensen's inequality, links between new and well-known fuzzy divergence measures were also created.

Keywords: *f -divergence; arithmetic-geometric mean divergence; harmonic mean divergence; relative arithmetic geometric divergence; theoretic exponential information distance measures; relative J-divergence.*

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1 Introduction

The fuzzy sets (FS) that Zadeh [1] described in 1965 have demonstrated useful applications in numerous academic domains. The concept of a fuzzy set is advantageous because it addresses ambiguity and uncertainty that the Cantorian set was unable to handle. According to fuzzy set theory, an element's membership in a fuzzy set [2] is expressed as a single value between zero and one. Nevertheless, since there may be some hesitation degree, it may not always be the case that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree. He developed a new theory to measure the uncertainty, which is also known as ambiguity, of a fuzzy set. If A be the subset of universe of discourse $i.e. X = \{x_1, \dots \dots, x_n\}$ then, A is defined as,

$$A = \{x_i/\mu_A(x_i): i = 1,2, \dots \dots \dots \dots, n\}.$$

Where $\mu_A(x_i)$ is a membership function and having the following properties:

1. If $\mu_A(x_i) = 0$, x_i does not belong to A and there is no ambiguity.
2. If $\mu_A(x_i) = 1$, x_i belong to A and there is no ambiguity.
3. If $\mu_A(x_i) = 0.5$, there is maximum ambiguity whether x_i belong to A or not.

Later, in 1983, the concept of intuitionistic fuzziness was first time introduced by Atanassov, Krassimir T. [3,4] in his original paper Intuitionistic Fuzzy Set (IFS). He developed a new theory to measure the uncertainty. According to him, if F be a fixed set then an intuitionistic fuzzy set S in F is an object having the form

$$S = \{< x, \mu_S(x), \nu_S(x) >/x \in F\}.$$

Where the function $\mu_S(x)$ and $\nu_S(x)$ define the degree of membership and degree of non-membership of the element $x \in S$ to $S \subset F$ respectively.

The function $\mu_S(x)$ and $\nu_S(x)$ satisfy the following condition.

$$(\forall\, x \in F)\ (0 \le \mu_S(x) + \nu_S(x) \le 1).$$

Obviously, fuzzy set has the form $\{< x, \mu_S(x), 1 - \mu_S(x) >/x \in F\}$.

A measure of fuzziness $f\big(\mu_S(x), \nu_S(x)\big)$ is an Intuitionistic fuzzy set should have atleast the following conditions:

(C_1) It should be continuous in this range of $(0 \le \mu_S(x_i) + \nu_S(x_i) \le 1), (i = 1, \dots \dots, n)$.

(C_2) It should be zero when $\mu_S(x_i) = 0$ and $\nu_S(x_i) = 0$.

(C_3) It should be not changed when any of $\mu_S(x_i)$ is changed into $\nu_S(x_i)$.

(C_4) It should be defined for all $\mu_S(x_i)$ and $\nu_S(x_i)$ $(i = 1, \dots \dots, n)$ in the range of $(0 \le \mu_S(x_i) +$

$$\nu_S(x_i) \le 1), (i = 1, \dots \dots, n).$$

(C_5) It should be maximum when $\mu_S(x_i) = \frac{1}{2}$ and $\nu_S(x_i) = \frac{1}{2}$ $(i = 1, \dots \dots, n)$.

(C_6) It should be increasing function of $\mu_S(x_i)$ when $0 \le \mu_S(x_i) \le \frac{1}{2}$ and decreasing function of $\mu_S(x_i)$ when $\frac{1}{2} \le \mu_S(x_i) \le 1$ and other variable are kept fixed. It should be decreasing funcion of $\nu_S(x_i)$ when $0 \le \nu_S(x_i) \le \frac{1}{2}$ and increasing function of $\nu_S(x_i)$ when $\frac{1}{2} \le \nu_S(x_i) \le 1$and other variable are kept fixed.

(C_7) It should be concave function of $\mu_S(x_i)$, when $\nu_S(x_i)$ set as a constant.

Using the f-divergence functional, Csiszar [5,6] established a generalized measure of information in 1961.

$$I_f(P,Q) = \sum_{i=1}^{n} q_i f\left(\frac{p_i}{q_i}\right) \tag{1.1}$$

Where $f: \boldsymbol{R}_+ \rightarrow \boldsymbol{R}_+$ is a convex function and $P, Q \in \Gamma n$. Here

$$\Gamma n = \left\{P = (p_1, p_2, \dots \dots \dots, p_n) \middle| p_i \geq 0, \sum_{i=1}^{n} p_i = 1\right\}, n \geq 2$$

be the collection of all discrete probability distributions with finite lengths. The literature on information theory and statistics includes a wide variety of information and divergence measures. Here, we'll provide a few examples of Csiszar f-divergence measure [7,8] and there corresponding intuitionistic fuzzy relative Information [9]

$$K(A,B) = \sum_{i=1}^{n}\left[\mu_A(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)} + \nu_A(x_i)\log\frac{\nu_A(x_i)}{\nu_B(x_i)}\right] \tag{1.2}$$

Intuitionistic fuzzy Chi-square divergence [10]

$$\mathcal{X}^2(A,B) = \sum_{i=1}^{n}\left(\left(\frac{\mu_A^2(x_i)}{\mu_B(x_i)} - \mu_B(x_i)\right) + \left(\frac{\nu_A^2(x_i)}{\nu_B(x_i)} - \nu_B(x_i)\right)\right) \tag{1.3}$$

Intuitionistic fuzzy relative J-S divergence [9]

$$F(A,B) = \sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right) + \sum_{i=1}^{n}\nu_A(x_i)\log\left(\frac{2\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right) \tag{1.4}$$

Intuitionistic fuzzy Hellinger discrimination [11]

$$h(P,Q) = 1 - B(P,Q) = \frac{1}{2}\sum_{i=1}^{n}\left(\sqrt{p_i} - \sqrt{q_i}\right)^2 \tag{1.5}$$

Where $B(A,B) = \sum_{i=1}^{n}\left(\sqrt{\frac{\mu_A(x_i)}{\mu_B(x_i)}} + \sqrt{\frac{\nu_A(x_i)}{\nu_B(x_i)}}\right)$ is known as Bhattacharya distance measure [12]

Intuitionistic fuzzy Verma [13] distance measure (Motivated by Kullback and Liebler [14])

$$V_a(A,B) = \sum_{i=1}^{n}\mu_B(x_i)\ln\left(\frac{\mu_B(x_i)+a\mu_A(x_i)}{\mu_A(x_i)}\right) + \sum_{i=1}^{n}\nu_B(x_i)\ln\left(\frac{\nu_B(x_i)+a\nu_A(x_i)}{\nu_A(x_i)}\right)$$
$$-\left(\mu_A(x_i) + \nu_A(x_i)\right)\ln(1+a), \quad a > 0 \tag{1.6}$$

and another intuitionistic fuzzy Verma [15] distance measure, due to general rule, is given by

$$V_a(A,B) = \sum_{i=1}^{n}\left(\ln\left(\frac{1+a\mu_A(x_i)}{1+a\mu_B(x_i)}\right) - \ln\frac{\mu_A(x_i)}{\mu_B(x_i)}\right).\mu_B(x_i)\left(1 + a\mu_B(x_i)\right)$$

$$+\sum_{i=1}^{n}\left(\ln\left(\frac{1+a\nu_A(x_i)}{1+a\nu_B(x_i)}\right) - \ln\frac{\nu_A(x_i)}{\nu_B(x_i)}\right).\nu_B(x_i)\left(1 + a\nu_B(x_i)\right), \quad a > 0 \tag{1.7}$$

Intuitionistic fuzzy J-divergence measure [16,14]

$$J(A,B) = \sum_{i=1}^{n}\left(\mu_A(x_i) - \mu_B(x_i)\right)\log\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right) + \sum_{i=1}^{n}\left(\nu_A(x_i) - \nu_B(x_i)\right)\log\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}\right) \tag{1.8}$$

Intuitionistic fuzzy relative J-divergence measure [10,14]

$$J_R(A,B) = \sum_{i=1}^{n}\left(\mu_A(x_i) - \mu_B(x_i)\right)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_B(x_i)}\right)$$

$$+\sum_{i=1}^{n}\left(\nu_A(x_i)-\nu_B(x_i)\right)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_B(x_i)}\right) \tag{1.9}$$

Intuitionistic fuzzy relative Jensen-Shannon divergence measure [17]

$$F(A,B)=\sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)+\sum_{i=1}^{n}\nu_A(x_i)\log\left(\frac{2\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right) \tag{1.10}$$

and $$F(B,A)=\sum_{i=1}^{n}\mu_B(x_i)\log\left(\frac{2\mu_B(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)+\sum_{i=1}^{n}\nu_B(x_i)\log\left(\frac{2\nu_B(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right) \tag{1.11}$$

Intuitionistic fuzzy Jensen-Shannon [17] divergence measure

$$I(A,B)=\frac{1}{2}[F(B,A)+F(A,B)]=\frac{1}{2}\left[\sum_{i=1}^{n}\left(\mu_B(x_i)\log\left(\frac{2\mu_B(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)+\nu_B(x_i)\log\left(\frac{2\nu_B(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)\right)\right]$$

$$-\frac{1}{2}\left[\sum_{i=1}^{n}\left(\mu_A(x_i)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_A(x_i)}\right)+\nu_A(x_i)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}\right)\right)\right] \tag{1.12}$$

Intuitionistic fuzzy arithmetic-geometric mean divergence [9,18]

$$T(A,B)=\sum_{i=1}^{n}\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2}\right)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\sqrt{\mu_A(x_i).\mu_B(x_i)}}\right)+\sum_{i=1}^{n}\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2}\right)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\sqrt{\nu_A(x_i).\nu_B(x_i)}}\right) \tag{1.13}$$

Intuitionistic fuzzy arithmetic mean divergence [19,18]

$$\boldsymbol{A}(A,B)=\sum_{i=1}^{n}\left[\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2}\right)+\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2}\right)\right] \tag{1.14}$$

Intuitionistic fuzzy harmonic mean divergence [19,18]

$$H(A,B)=\sum_{i=1}^{n}\left[\left(\frac{2\mu_A(x_i).\mu_B(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)+\left(\frac{2\nu_A(x_i).\nu_B(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)\right] \tag{1.15}$$

Intuitionistic fuzzy relative arithmetic-geometric divergence [9]

$$G(B,A)=\sum_{i=1}^{n}\left[\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2}\right)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_B(x_i)}\right)+\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2}\right)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_B(x_i)}\right)\right] \tag{1.16}$$

Intuitionistic fuzzy triangular discrimination divergence measure

$$\Delta_1(A,B)=\sum_{i=1}^{n}\left(\frac{\left(\mu_A(x_i)-\mu_B(x_i)\right)^3}{\mu_A(x_i)+\mu_B(x_i)}+\frac{\left(\nu_A(x_i)-\nu_B(x_i)\right)^3}{\nu_A(x_i)+\nu_B(x_i)}\right) \tag{1.17}$$

Intuitionistic fuzzy logarithmic mean divergence [19]

$$L(A,B)=\sum_{i=1}^{n}\left(\frac{\mu_A(x_i)-\mu_B(x_i)}{\log\mu_A(x_i)-\log\mu_B(x_i)}+\frac{\mu_A(x_i)-\mu_B(x_i)}{\log\mu_A(x_i)-\log\mu_B(x_i)}\right) \tag{1.18}$$

Intuitionistic fuzzy Kumar-Johnson [20] distance measures

$$\psi M(A,B)=\frac{1}{2}\sum_{i=1}^{n}\left(\frac{\left(\mu_A^2(x_i)-\mu_B^2(x_i)\right)^2}{\left(\mu_A(x_i).\mu_B(x_i)\right)^{3/2}}+\frac{\left(\nu_A^2(x_i)-\nu_B^2(x_i)\right)^2}{\left(\nu_A(x_i).\nu_B(x_i)\right)^{3/2}}\right) \tag{1.19}$$

Intuitionistic fuzzy theoretic exponential information distance measures

$$\phi D(A,B)=\frac{1}{2}\sum_{i=1}^{n}\left(\frac{\left(\mu_A^2(x_i)-\mu_B^2(x_i)\right)^2}{\left(\mu_A(x_i)\mu_B(x_i)\right)^{3/2}}.e^{\frac{\mu_A(x_i)}{\mu_B(x_i)}}+\frac{\left(\nu_A^2(x_i)-\nu_B^2(x_i)\right)^2}{\left(\nu_A(x_i)\nu_B(x_i)\right)^{3/2}}.e^{\frac{\nu_A(x_i)}{\nu_B(x_i)}}\right) \tag{1.20}$$

and

$$\phi D_\rho(A,B)=\frac{1}{4}\sum_{i=1}^{n}\left[\begin{array}{c}\frac{(\mu_A(x_i)-\mu_B(x_i))\left(\mu_A^2(x_i)-\mu_B^2(x_i)\right)\left(2\mu_A^3(x_i)+5\mu_A^2(x_i)\mu_B(x_i)-2\mu_A(x_i)\mu_B^2(x_i)+3\mu_B^3(x_i)\right)}{\mu_B(x_i)(\mu_A(x_i)\mu_B(x_i))^{5/2}}.e^{\frac{\mu_A(x_i)}{\mu_B(x_i)}}\\ \frac{(\nu_A(x_i)-\nu_B(x_i))\left(\nu_A^2(x_i)-\nu_B^2(x_i)\right)\left(2\nu_A^3(x_i)+5\nu_A^2(x_i)\nu_B(x_i)-2\nu_A(x_i)\nu_B^2(x_i)+3\nu_B^3(x_i)\right)}{\nu_B(x_i)(\nu_A(x_i)\nu_B(x_i))^{5/2}}.e^{\frac{\nu_A(x_i)}{\nu_B(x_i)}}\end{array}\right] \tag{1.21}$$

2 Our Work

Proposition 2.1 Let $(A,B)\in \Gamma n\times \Gamma n$ and if $K(B,A)$, $F(A,B)$, $J(A,B)$ shows K-L, Relative J-S, J-divergence measures respectively. Then we have the following new equality

$$2D_1(A,B)=J(A,B)-K(B,A)+F(B,A)-F(A,B).$$

Proof: If we take the convex function,

$$F(y)=\frac{(y-1)^2}{(y+1)^1}+\frac{1}{3}\frac{(y-1)^4}{(y+1)^3}+\frac{1}{5}\frac{(y-1)^6}{(y+1)^5}+\frac{1}{7}\frac{(y-1)^8}{(y+1)^7}+\cdots$$

$$=(y-1)\left[\left(\frac{y-1}{y+1}\right)^1+\frac{1}{3}\left(\frac{y-1}{y+1}\right)^3+\frac{1}{5}\left(\frac{y-1}{y+1}\right)^5+\frac{1}{7}\left(\frac{y-1}{y+1}\right)^7+\cdots\right]$$

$$i.e.\qquad F(y)=(y-1)\frac{1}{2}\log\left[\frac{1+\left(\frac{y-1}{y+1}\right)}{1-\left(\frac{y-1}{y+1}\right)}\right]=(y-1)\frac{1}{2}\log y \tag{2.1.1}$$

Next we get the subsequent f-divergence measure if we put $y=\frac{p_i}{q_i}$ in (2.1.1) $i.e.$

$$D_1(P,Q)=\sum_{i=1}^{n}q_i\left(\frac{p_i}{q_i}-1\right)\frac{1}{2}\log\left(\frac{p_i}{q_i}\right).$$

Applying intuitionistic fuzzy in above equation, we achieve

$$D_1(A,B)=\sum_{i=1}^{n}\mu_B(x_i)\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}-1\right)\frac{1}{2}\log\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)$$

$$+\sum_{i=1}^{n}\nu_B(x_i)\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}-1\right)\frac{1}{2}\log\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}\right)$$

$$i.e.\quad 2D_1(A,B)=\sum_{i=1}^{n}\left(\mu_A(x_i)-\mu_B(x_i)\right)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}$$

$$+\sum_{i=1}^{n}\left(\nu_A(x_i)-\nu_B(x_i)\right)\log\frac{\nu_A(x_i)}{\nu_B(x_i)}$$

$$i.e.\quad 2D_1(A,B)=\sum_{i=1}^{n}\mu_A(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}-\sum_{i=1}^{n}\mu_B(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}$$

$$+\sum_{i=1}^{n}\nu_A(x_i)\log\frac{\nu_A(x_i)}{\nu_B(x_i)}-\sum_{i=1}^{n}\nu_B(x_i)\log\frac{\nu_A(x_i)}{\nu_B(x_i)}$$

$$=\sum_{i=1}^{n}\left(\mu_A(x_i)-\mu_B(x_i)+\mu_B(x_i)\right)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}-\sum_{i=1}^{n}\mu_B(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}$$

$$+\sum_{i=1}^{n}\left(\nu_A(x_i)-\nu_B(x_i)+\nu_B(x_i)\right)\log\frac{\nu_A(x_i)}{\nu_B(x_i)}-\sum_{i=1}^{n}\nu_B(x_i)\log\frac{\nu_A(x_i)}{\nu_B(x_i)}$$

$$=\sum_{i=1}^{n}\left(\mu_A(x_i)-\mu_B(x_i)\right)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}+\sum_{i=1}^{n}\mu_B(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}-\sum_{i=1}^{n}\mu_B(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}$$

$$+\sum_{i=1}^{n}\left(\nu_A(x_i)-\nu_B(x_i)\right)\log\frac{\nu_A(x_i)}{\nu_B(x_i)}+\sum_{i=1}^{n}\nu_B(x_i)\log\frac{\nu_A(x_i)}{\nu_B(x_i)}-\sum_{i=1}^{n}\nu_B(x_i)\log\frac{\nu_A(x_i)}{\nu_B(x_i)}$$

$$=J(A,B)-\sum_{i=1}^{n}\mu_B(x_i)\log\frac{\mu_B(x_i)}{\mu_A(x_i)}-\sum_{i=1}^{n}\mu_B(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}$$

$$-\sum_{i=1}^{n}\nu_B(x_i)\log\frac{\nu_B(x_i)}{\nu_A(x_i)}-\sum_{i=1}^{n}\nu_B(x_i)\log\frac{\nu_A(x_i)}{\nu_B(x_i)}$$

$$=J(A,B)-K(B,A)-\sum_{i=1}^{n}\mu_B(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}-\sum_{i=1}^{n}\nu_B(x_i)\log\frac{\nu_A(x_i)}{\nu_B(x_i)}$$

$$=J(A,B)-K(B,A)-\sum_{i=1}^{n}\mu_B(x_i)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_B(x_i)}.\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)$$

$$-\sum_{i=1}^{n}\nu_B(x_i)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_B(x_i)}.\frac{2\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)$$

$$=J(A,B)-K(B,A)+\sum_{i=1}^{n}\mu_B(x_i)\log\left(\frac{2\mu_B(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)-\sum_{i=1}^{n}\mu_B(x_i)\log\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)$$

$$+\sum_{i=1}^{n}\nu_B(x_i)\log\left(\frac{2\nu_B(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)-\sum_{i=1}^{n}\nu_B(x_i)\log\left(\frac{2\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)$$

$$=J(A,B)-K(B,A)+F(B,A)-\sum_{i=1}^{n}\frac{\mu_B(x_i)}{\mu_A(x_i)}\left(\mu_A(x_i)\log\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)\right)$$

$$-\sum_{i=1}^{n}\frac{\nu_B(x_i)}{\nu_A(x_i)}\left(\nu_A(x_i)\log\left(\frac{2\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)\right)$$

$$=J(A,B)-K(B,A)+F(B,A)-F(A,B).$$

Hence the required equality.

Proposition 2.2 Let $(A,B)\in\Gamma n\times\Gamma n$ and if $\boldsymbol{A}(A,B)$, $G(A,B)$, $F(A,B)$, $\mathcal{X}^2(A,B)$ represents AMD, RAGD, RJSD, Chi-Square distance measures respectively then show that the new equality relation

$$\frac{4D_1(P,Q).\boldsymbol{A}(A,B)-G(P,Q)-G(Q,P)}{A(P,Q)}=F(Q,P)+F(P,Q)$$

and inequality relation

$$4D_1(P,Q).\boldsymbol{A}(A,B)-G(P,Q)-G(Q,P)<[\mathcal{X}^2(Q,P)+\mathcal{X}^2(P,Q)]\boldsymbol{A}(A,B)$$

Proof: If we put $y=\frac{p_i+q_i}{2q_i}$ in the convex function (2.1.1) then we get the subsequent f-divergence measure

$$D_1(P,Q)=(y-1)\frac{1}{2}\log y=\left(\frac{p_i+q_i}{2q_i}-1\right)\frac{1}{2}\log\left(\frac{p_i+q_i}{2q_i}\right)$$
$$2D_1(P,Q)=\frac{1}{2}\sum_{i=1}^{n}(p_i-q_i)\log\left(\frac{p_i+q_i}{2q_i}\right).$$

Applying intuitionistic fuzzy in above equation, we achieve

$$D_1(A,B)=\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_B(x_i)}-1\right)\frac{1}{2}\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_B(x_i)}\right)+\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}-1\right)\frac{1}{2}\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}\right)$$

$$i.e.\ \ 2D_1(A,B)=\frac{1}{2}\sum_{i=1}^{n}\left(\mu_A(x_i)-\mu_B(x_i)\right)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_B(x_i)}\right)$$

$$+\frac{1}{2}\sum_{i=1}^{n}\left(\nu_A(x_i)-\nu_B(x_i)\right)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}\right)$$

$$i.e.\ \ 2D_1(A,B)=\frac{1}{2}\left[\sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_B(x_i)}\right)-\sum_{i=1}^{n}\mu_B(x_i)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_B(x_i)}\right)\right]$$

$$+\frac{1}{2}\left[\sum_{i=1}^{n}\nu_A(x_i)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}\right)-\sum_{i=1}^{n}\nu_B(x_i)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}\right)\right]$$

$$i.e.\ \ 2D_1(A,B)=\frac{1}{2}\left[\begin{array}{c}\sum_{i=1}^{n}\mu_B(x_i)\log\left(\frac{2\mu_B(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)-\sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{2\mu_B(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)+\\ \sum_{i=1}^{n}\nu_B(x_i)\log\left(\frac{2\nu_B(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)-\sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{2\nu_B(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)+\\ \sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_A(x_i)}\right)-\sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_A(x_i)}\right)+\\ \sum_{i=1}^{n}\nu_A(x_i)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}\right)-\sum_{i=1}^{n}\nu_A(x_i)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}\right)\end{array}\right]$$

$$i.e.\ \ 2D_1(A,B)=\frac{1}{2}\left[\sum_{i=1}^{n}\mu_B(x_i)\log\left(\frac{2\mu_B(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)-\sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_A(x_i)}\right)\right]+$$

$$\frac{1}{2}\left[\sum_{i=1}^{n}\nu_B(x_i)\log\left(\frac{2\nu_B(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)-\sum_{i=1}^{n}\nu_A(x_i)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}\right)\right]$$

$$+\frac{1}{2}\left[\sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_A(x_i)}\right)+\sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_B(x_i)}\right)\right]$$

$$+\frac{1}{2}\left[\sum_{i=1}^{n}\nu_A(x_i)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}\right)+\sum_{i=1}^{n}\nu_A(x_i)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_B(x_i)}\right)\right]$$

$$i.e.\ \ \ 2D_1(A,B)=\frac{1}{2}[F(B,A)+F(A,B)]+\frac{1}{2}\left[\sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_A(x_i)}.\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_B(x_i)}\right)\right]$$

$$+\frac{1}{2}\left[\sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}.\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_B(x_i)}\right)\right]$$

$$i.e.\ \ \ 2D_1(A,B)=\frac{1}{2}[F(B,A)+F(A,B)]+\frac{1}{2}\left[\sum_{i=1}^{n}\mu_A(x_i)\log\left(\frac{\left(\mu_A(x_i)+\mu_B(x_i)\right)^2}{4\mu_A(x_i).\mu_B(x_i)}\right)\right]$$

$$+\frac{1}{2}\left[\sum_{i=1}^{n}\nu_A(x_i)\log\left(\frac{\left(\nu_A(x_i)+\nu_B(x_i)\right)^2}{4\nu_A(x_i).\nu_B(x_i)}\right)\right]$$

$$=\frac{1}{2}[F(B,A)+F(A,B)]+\left(\sum_{i=1}^{n}\mu_A(x_i)\right)\left(\sum_{i=1}^{n}\frac{2}{\mu_A(x_i)+\mu_B(x_i)}\right)$$

$$\left(\sum_{i=1}^{n}\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2}\right)\log\left(\frac{\left(\mu_A(x_i)+\mu_B(x_i)\right)^2}{4\mu_A(x_i).\mu_B(x_i)}\right)\right)+\left(\sum_{i=1}^{n}\nu_A(x_i)\right)\left(\sum_{i=1}^{n}\frac{2}{\nu_A(x_i)+\nu_B(x_i)}\right)$$

$$\left(\sum_{i=1}^{n}\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2}\right)\log\left(\frac{\left(\nu_A(x_i)+\nu_B(x_i)\right)^2}{4\nu_A(x_i).\nu_B(x_i)}\right)\right)$$

$$=\frac{1}{2}[F(B,A)+F(A,B)]$$

$$+\frac{1}{2A(A,B)}\left[\sum_{i=1}^{n}\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2}\right)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_A(x_i)}\right)+\sum_{i=1}^{n}\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2}\right)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\mu_B(x_i)}\right)\right]$$

$$+\frac{1}{2A(A,B)}\left[\sum_{i=1}^{n}\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2}\right)\log\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2\nu_A(x_i)}\right)+\sum_{i=1}^{n}\left(\frac{\nu_A(x_i)+\nu_B(x_i)}{2}\right)\log\left(\frac{\mu_A(x_i)+\mu_B(x_i)}{2\nu_B(x_i)}\right)\right]$$

Thus, $$2D_1(A,B) = \frac{1}{2}[F(B,A) + F(A,B)] + \frac{1}{2\boldsymbol{A}(A,B)}[G(A,B) + G(B,A)]$$

$$i.e.\quad \frac{4D_1(A,B).\boldsymbol{A}(A,B) - G(A,B) - G(B,A)}{\boldsymbol{A}(A,B)} = F(B,A) + F(A,B) \tag{2.2.1}$$

Which is the required equality.

But, $$F(B,A) + F(A,B) < \mathcal{X}^2(B,A) + \mathcal{X}^2(A,B)$$

From (2.2.1) we get the inequality

$$4D_1(A,B).\boldsymbol{A}(A,B) - G(A,B) - G(B,A) < [\mathcal{X}^2(B,A) + \mathcal{X}^2(A,B)]\boldsymbol{A}(A,B).$$

Hence the required result.

Proposition 2.3 Let $(A,B) \in \Gamma n \times \Gamma n$ and if $\mathcal{X}^2(A,B)$, $\psi M(A,B)$, $\phi D_\rho(A,B)$ shows chi-square, Kumar-Johnson, theoretic exponential information distance measures respectively then we have the following new inequality

$$\frac{1}{2}[\mathcal{X}^2(A,B) + \mathcal{X}^2(B,A)] \le \frac{1}{2}.\psi M(A,B) \le \phi D_\rho(A,B)$$

Proof: Ofcourse, $$\left(\sqrt{y} - 1\right)^2 \ge 0$$

$$i.e.\quad \sqrt{y} + \frac{1}{\sqrt{y}} \ge 2$$

$$i.e.\quad \frac{1}{2}\left(\sqrt{y} + \frac{1}{\sqrt{y}}\right) \ge 1$$

Obviously, $$\frac{1}{2}\left(\sqrt{y} + \frac{1}{\sqrt{y}}\right) \le \left[\frac{1}{2}\left(\sqrt{y} + \frac{1}{\sqrt{y}}\right)\right]^2 \le \frac{1}{4}(y+1).\frac{(2y^3 + 5y^2 - 2y + 3)}{y^2}.e^y$$

$$i.e.\quad \frac{1}{2}.\frac{(y-1)^2}{\sqrt{y}}\left(\sqrt{y} + \frac{1}{\sqrt{y}}\right) \le \frac{1}{4}.\frac{(y-1)^2}{\sqrt{y}}\left(\sqrt{y} + \frac{1}{\sqrt{y}}\right)^2 \le \frac{1}{4}(y+1).\frac{(y-1)^2}{\sqrt{y}}.\frac{(2y^3 + 5y^2 - 2y + 3)}{y^2}.e^y$$

$$i.e.\quad \frac{1}{2}.\frac{(y+1)(y-1)^2}{y} \le \frac{1}{4}.\frac{\left(y^2-1\right)^2}{y^{3/2}} \le \frac{1}{4}.\frac{(y-1)(y^2-1)(2y^3 + 5y^2 - 2y + 3)}{y^{5/2}}.e^y$$

Substituting y by $\frac{p_i}{q_i}$, we achieve the following result

$$\frac{1}{2}.\frac{\left(\frac{p_i}{q_i}+1\right)\left(\frac{p_i}{q_i}-1\right)^2}{\frac{p_i}{q_i}} \le \frac{1}{4}.\frac{\left[\left(\frac{p_i}{q_i}\right)^2 - 1\right]^2}{\left(\frac{p_i}{q_i}\right)^{3/2}} \le \frac{1}{4}.\frac{\left(\frac{p_i}{q_i}-1\right)\left(\left(\frac{p_i}{q_i}\right)^2 - 1\right)\left(2\left(\frac{p_i}{q_i}\right)^3 + 5\left(\frac{p_i}{q_i}\right)^2 - 2\frac{p_i}{q_i} + 3\right)}{\left(\frac{p_i}{q_i}\right)^{5/2}}.e^{p_i/q_i}$$

Applying intuitionistic fuzzy in above equation, we achieve

$$\frac{1}{2}.\left[\frac{\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}+1\right)\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}-1\right)^2}{\frac{\mu_A(x_i)}{\mu_B(x_i)}} + \frac{\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}+1\right)\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}-1\right)^2}{\frac{\nu_A(x_i)}{\nu_B(x_i)}}\right] \le \frac{1}{4}.\left[\frac{\left[\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)^2 - 1\right]^2}{\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)^{3/2}} + \frac{\left[\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}\right)^2 - 1\right]^2}{\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}\right)^{3/2}}\right]$$

$$\leq \frac{1}{4}.\left[\begin{array}{c}\dfrac{\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}-1\right)\left(\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)^2-1\right)\left(2\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)^3+5\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)^2-2\frac{\mu_A(x_i)}{\mu_B(x_i)}+3\right)}{\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)^{5/2}}.e^{\mu_A(x_i)/\mu_B(x_i)} \\ \dfrac{\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}-1\right)\left(\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}\right)^2-1\right)\left(2\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}\right)^3+5\left(\frac{\nu_A(x_i)}{\nu_B(x_i)}\right)^2-2\frac{\nu_A(x_i)}{\nu_B(x_i)}+3\right)}{\left(\frac{\mu_A(x_i)}{\mu_B(x_i)}\right)^{5/2}}.e^{\nu_A(x_i)/\nu_B(x_i)}\end{array}\right]$$

$i.e.$

$$\frac{1}{2}\sum_{i=1}^{n}\left[\frac{(\mu_A(x_i)+\mu_B(x_i))(\mu_A(x_i)-\mu_B(x_i))^2}{\mu_A(x_i)(\mu_B(x_i))^2}+\frac{(\nu_A(x_i)+\nu_B(x_i))(\nu_A(x_i)-\nu_B(x_i))^2}{\nu_A(x_i)(\nu_B(x_i))^2}\right]$$

$$\leq \frac{1}{4}\sum_{i=1}^{n}\left[\frac{\left(\mu_A^2(x_i)-\mu_B^2(x_i)\right)^2}{\mu_B(x_i)(\mu_A(x_i)\mu_B(x_i))^{3/2}}+\frac{\left(\nu_A^2(x_i)-\nu_B^2(x_i)\right)^2}{\nu_B(x_i)(\nu_A(x_i).\nu_B(x_i))^{3/2}}\right]$$

$$\leq \frac{1}{4}\sum_{i=1}^{n}\left[\begin{array}{c}\dfrac{(\mu_A(x_i)-\mu_B(x_i))\left(\mu_A^2(x_i)-\mu_B^2(x_i)\right)\left(2\mu_A^3(x_i)+5\mu_A^2(x_i)\mu_B(x_i)-2\mu_A(x_i)\mu_B^2(x_i)+3\mu_B^3(x_i)\right)}{\mu_B(x_i)(\mu_A(x_i)\mu_B(x_i))^{5/2}}.e^{\mu_A(x_i)/\mu_B(x_i)} \\ \dfrac{(\nu_A(x_i)-\nu_B(x_i))\left(\nu_A^2(x_i)-\nu_B^2(x_i)\right)\left(2\nu_A^3(x_i)+5\nu_A^2(x_i)\nu_B(x_i)-2\nu_A(x_i)\nu_B^2(x_i)+3\nu_B^3(x_i)\right)}{\nu_B(x_i)(\nu_A(x_i)\nu_B(x_i))^{5/2}}.e^{\nu_A(x_i)/\nu_B(x_i)}\end{array}\right]$$

$i.e.$

$$\frac{1}{2}\sum_{i=1}^{n}\left[\frac{(\mu_A(x_i)+\mu_B(x_i))(\mu_A(x_i)-\mu_B(x_i))^2}{\mu_A(x_i)\mu_B(x_i)}+\frac{(\nu_A(x_i)+\nu_B(x_i))(\nu_A(x_i)-\nu_B(x_i))^2}{\nu_A(x_i)\nu_B(x_i)}\right]$$

$$\leq \frac{1}{4}\sum_{i=1}^{n}\left[\frac{\left(\mu_A^2(x_i)-\mu_B^2(x_i)\right)^2}{(\mu_A(x_i)\mu_B(x_i))^{3/2}}+\frac{\left(\nu_A^2(x_i)-\nu_B^2(x_i)\right)^2}{(\nu_A(x_i).\nu_B(x_i))^{3/2}}\right]$$

$$\leq \frac{1}{4}\sum_{i=1}^{n}\left[\begin{array}{c}\dfrac{(\mu_A(x_i)-\mu_B(x_i))\left(\mu_A^2(x_i)-\mu_B^2(x_i)\right)\left(2\mu_A^3(x_i)+5\mu_A^2(x_i)\mu_B(x_i)-2\mu_A(x_i)\mu_B^2(x_i)+3\mu_B^3(x_i)\right)}{(\mu_A(x_i)\mu_B(x_i))^{5/2}}.e^{\mu_A(x_i)/\mu_B(x_i)} \\ +\dfrac{(\nu_A(x_i)-\nu_B(x_i))\left(\nu_A^2(x_i)-\nu_B^2(x_i)\right)\left(2\nu_A^3(x_i)+5\nu_A^2(x_i)\nu_B(x_i)-2\nu_A(x_i)\nu_B^2(x_i)+3\nu_B^3(x_i)\right)}{(\nu_A(x_i)\nu_B(x_i))^{5/2}}.e^{\nu_A(x_i)/\nu_B(x_i)}\end{array}\right]$$

$i.e.$

$$\frac{1}{2}\left[\sum_{i=1}^{n}\left(\left(\frac{\mu_A^2(x_i)}{\mu_B(x_i)}-\mu_B(x_i)\right)+\left(\frac{\nu_A^2(x_i)}{\nu_B(x_i)}-\nu_B(x_i)\right)\right)\right]$$

$$+\frac{1}{2}\left[\sum_{i=1}^{n}\left(\left(\frac{\mu_B^2(x_i)}{\mu_A(x_i)}-\mu_A(x_i)\right)+\left(\frac{\nu_B^2(x_i)}{\nu_A(x_i)}-\nu_A(x_i)\right)\right)\right]$$

$$\leq \frac{1}{4}\sum_{i=1}^{n}\left(\frac{\left(\mu_A^2(x_i)-\mu_B^2(x_i)\right)^2}{(\mu_A(x_i)\mu_B(x_i))^{3/2}}+\frac{\left(\nu_A^2(x_i)-\nu_B^2(x_i)\right)^2}{(\nu_A(x_i)\nu_B(x_i))^{3/2}}\right)$$

$$\leq \frac{1}{4}\sum_{i=1}^{n}\left[\begin{array}{c}\dfrac{(\mu_A(x_i)-\mu_B(x_i))\left(\mu_A^2(x_i)-\mu_B^2(x_i)\right)\left(2\mu_A^3(x_i)+5\mu_A^2(x_i)\mu_B(x_i)-2\mu_A(x_i)\mu_B^2(x_i)+3\mu_B^3(x_i)\right)}{(\mu_A(x_i)\mu_B(x_i))^{5/2}}.e^{\mu_A(x_i)/\mu_B(x_i)} \\ +\dfrac{(\nu_A(x_i)-\nu_B(x_i))\left(\nu_A^2(x_i)-\nu_B^2(x_i)\right)\left(2\nu_A^3(x_i)+5\nu_A^2(x_i)\nu_B(x_i)-2\nu_A(x_i)\nu_B^2(x_i)+3\nu_B^3(x_i)\right)}{(\nu_A(x_i)\nu_B(x_i))^{5/2}}.e^{\nu_A(x_i)/\nu_B(x_i)}\end{array}\right]$$

$i.e.$

$$\frac{1}{2}[\mathcal{X}^2(A,B)+\mathcal{X}^2(B,A)]\leq \frac{1}{2}.\psi M(A,B)\leq \phi D_\rho(A,B).$$

Hence the inequality.

Proposition 2.4 Let $(P,Q)\in \Gamma n\times \Gamma n$ and if $B(P,Q)$, $H(P,Q)$, $\mathcal{X}^2(P,Q)$ shows Bhattacharya, Harmonic Mean, Chi-Square distance measures respectively then show that the following new inequality

$$\mathcal{X}^2(A,B) > B(A,B) > H(A,B)$$

Proof: Considering an inequality

$$y+1 > \sqrt{y} > \frac{y}{y+1}$$

obviously $$y^2-1 > \sqrt{y} > \frac{2y}{y+1}.$$

Substituting y by $\frac{p_i}{q_i}$, we achieve the following result

$$\left(\frac{{p_i}^2}{{q_i}^2}-1\right) > \sqrt{{p_i}/{q_i}} > \frac{2^{p_i}/q_i}{{p_i}/{q_i}+1}$$

Applying intuitionistic fuzzy in above equation, we achieve as follows:

$$\left(\frac{\mu_A^2(x_i)}{\mu_B^2(x_i)}-1\right)+\left(\frac{\nu_A^2(x_i)}{\nu_B^2(x_i)}-1\right) > \sqrt{\frac{\mu_A(x_i)}{\mu_B(x_i)}}+\sqrt{\frac{\nu_A(x_i)}{\nu_B(x_i)}} > \frac{2.\frac{\mu_A(x_i)}{\mu_B(x_i)}}{\frac{\mu_A(x_i)}{\mu_B(x_i)}+1}+\frac{2.\frac{\nu_A(x_i)}{\nu_B(x_i)}}{\frac{\nu_A(x_i)}{\nu_B(x_i)}+1}$$

$$i.e.\ \left(\frac{\mu_A^2(x_i)}{\mu_B^2(x_i)}-1\right)+\left(\frac{\nu_A^2(x_i)}{\nu_B^2(x_i)}-1\right) > \sqrt{\frac{\mu_A(x_i)}{\mu_B(x_i)}}+\sqrt{\frac{\nu_A(x_i)}{\nu_B(x_i)}} > \frac{2.\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}+\frac{2.\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}$$

$$i.e.\ \sum_{i=1}^n\left[\mu_B(x_i)\left(\frac{\mu_A^2(x_i)}{\mu_B^2(x_i)}-1\right)+\nu_B(x_i)\left(\frac{\nu_A^2(x_i)}{\nu_B^2(x_i)}-1\right)\right]$$

$$> \sum_{i=1}^n\left[\mu_B(x_i)\sqrt{\frac{\mu_A(x_i)}{\mu_B(x_i)}}+\nu_B(x_i)\sqrt{\frac{\nu_A(x_i)}{\nu_B(x_i)}}\right]$$

$$> \sum_{i=1}^n\left[\mu_B(x_i)\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)+\nu_B(x_i)\left(\frac{2.\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)\right]$$

$$i.e.\ \sum_{i=1}^n\left[\mu_B(x_i)\left(\frac{\mu_A^2(x_i)}{\mu_B^2(x_i)}-1\right)+\nu_B(x_i)\left(\frac{\nu_A^2(x_i)}{\nu_B^2(x_i)}-1\right)\right]$$

$$> \sum_{i=1}^n\left[\sqrt{\mu_A(x_i).\mu_B(x_i)}+\sqrt{\nu_A(x_i).\nu_B(x_i)}\right]$$

$$> \sum_{i=1}^n\left[\mu_B(x_i)\left(\frac{2\mu_A(x_i)}{\mu_A(x_i)+\mu_B(x_i)}\right)+\nu_B(x_i)\left(\frac{2.\nu_A(x_i)}{\nu_A(x_i)+\nu_B(x_i)}\right)\right]$$

$$i.e. \quad \mathcal{X}^2(A,B) > B(A,B) > H(A,B)$$

Hence the desired result.

3 Conclusions

Using fuzzy Csiszar's f-divergence and fuzzy new f-divergence measure properties with the demonstration of validity, we have achieved some series of fuzzy divergence measures in this study. We have suggested a generalized series of Kulback-Leibler, arithmetic divergence measures, etc. combinations. Inequalities on both new and well-known fuzzy divergence metrics have also been derived by us.

Competing Interests

Author has declared that no competing interests exist.

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