Al-Azhar Bull. Sci. Vol. 24, No. 1 (June.): pp. 65-72, 2013.

MODEL OF REFERENCE ORBIT FOR SATELLITES CONSTELLATION

O. S. ALI[1]; M.N. ISMAIL[1]; A. BAKRY[1]; I.A. HASSAN[1]; NADIA A. SAAD[2] ; B. GIOVANNI[3]

1. *Al-Azhar University, Faculty of Science*
2. *National Research Institutes of Astronomy and Geophysics (NRIAG)*
3. *Aerospace Engineering, Roma Univ. Italy*

Abstract

The current work will be focused upon the study of the effects of the earth's gravitational field on the motion of an artificial satellite constellation and a computational model for this perturbation on the constellation orbit will be done; the earth's gravitational field up to second zonal harmonics is considered. This perturbation make a drift for the orbital elements of satellite constellation that make continues coverage and lead to a shift of the whole constellation structure. The equations of motion under effects of oblatenees force are solved numerically by using Rung-Kotta method. The results show that this model has efficiency to compute the perturbations for satellite constellation which already congruent with observing data.

1. Introduction

With the continual development of astronautical technology, small satellites have attracted more and more attention of the public. This is because they are small, light and inexpensive, their period of development is short, and they can be launched conveniently (either individually, or as pick-ups, or several small satellites may be launched with one rocket). Constellation of medium-small satellites (sometimes also known as network constellation or cluster of satellites) is an important subject of research in the application of small satellites.

A constellation is formed by a number of satellites, which have basically the same orbital elements (semi-major axes, inclinations and eccentricities) and close latitudes and nodal longitudes. The constellations of small satellites have a wide and practical significance, and one of its applications is the positioning of locations on the earth's surface.

When defining the same reference orbit for all the satellites of the constellation, the difficulty is to choose a realistic dynamical model which is independent of the initial conditions of each satellite. A dynamical model which includes only the main zonal deformations of the Earth gravity field can provide the desired reference orbit.

According to Kaula's theory (Kaula, 1966), these coefficients produce three types of orbital perturbations on a satellite moving around the Earth:

1- Secular perturbations, due to the even coefficients, on the ascending node, on the argument of perigee and on the mean anomaly, 2- Long-period perturbations, mainly due to the odd coefficients, on all the orbital elements except on the semi-major axis at the revolution period of the perigee and at the sub-multiples of this period,

3- Short-period perturbations on all the orbital elements at the revolution period of the satellite around the Earth, the mean anomaly, and at the sub-multiples of this period.

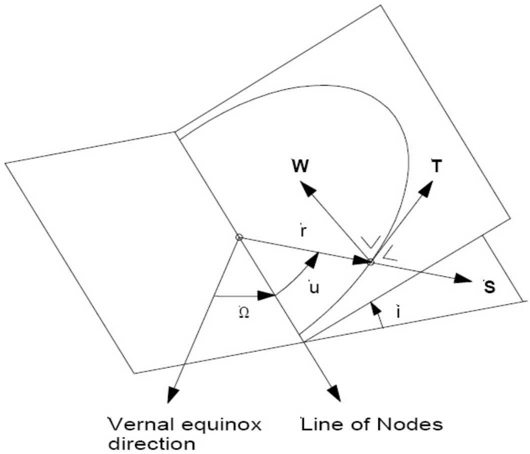
Finally the phasing of the orbit can be first executed by only considering the secular variations of the mean parameters.

Some forces which are difficult to forecast, like the drag, the radiation pressure and also the thermal effects, are usually not included into the dynamical model of the reference orbit. The most convenient solution in that case is to calculate the same reference orbit for all the satellites of the constellation. To respect such a definition of the reference orbit, only zonal coefficients of the Earth gravity field have to be included in the dynamical model. This orbit is firstly calculated in terms of mean parameters numerically in an iterative process where the cycle duration, the semi-major axis and the eccentricity of the orbit are recomputed each time. We obtain a frozen and phased orbit very stable over every cycle.

The final reference orbit preserves the stability properties of the mean reference orbit.

To solve the problem in this work, its more convenient to use Gauss form of Lagrange's equations which defined as,

$$\left.\begin{aligned}
\frac{da}{dt} &= \frac{2e\sin f}{n\sqrt{1-e^2}}S + \frac{2a\sqrt{1-e^2}}{nr}T \\
\frac{de}{dt} &= \frac{\sqrt{1-e^2}\sin f}{na}S + \frac{\sqrt{1-e^2}}{nae}\left[\frac{a(1-e^2)}{r} - \frac{r}{a}\right]T \\
\frac{di}{dt} &= \frac{r\cos(f+\omega)}{na^2\sqrt{1-e^2}}W \\
\frac{d\Omega}{dt} &= \frac{r\sin(f+\omega)}{na^2\sqrt{1-e^2}\sin i} \\
\frac{d\omega}{dt} &= -\dot{\Omega}\cos i - \frac{\sqrt{1-e^2}}{nae}\left[\cos f.s - \sin f\left(1+\frac{r}{a(1-e^2)}\right)T\right] \\
\frac{dM}{dt} &= n - \frac{2}{na^2}Sr - \dot{\omega}\sqrt{1-e^2} - \dot{\Omega}\sqrt{1-e^2}\cos i
\end{aligned}\right\} \quad (1)$$



The three components of the perturbation force are:

$$S = -\frac{\partial R}{\partial r}, \quad T = -\frac{1}{r}\frac{\partial R}{\partial u}, \quad W = \frac{-1}{r \sin u}\frac{\partial R}{\partial i}$$

where
S: along the instantaneous radius vector,
T: perpendicular to the instantaneous radius vector in the direction of the motion,
W: normal to the osculating plane of the orbit.

2. Formulation:

The Earth gravitational potential expressed as

$$V = \frac{-\mu}{r}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^{n} P_n^m(\sin\delta)[C_{nm}\cos m\lambda + S_{nm}\sin n\lambda] \quad (2)$$

$\mu \cong GM_e$ is the product of the Earth's gravitational constant and the mass of the earth, ($398600.8\, Km^3/\sec^2$)

G is the universal constant of gravity,

M_e is the mass of the Earth,

R is the equatorial radius of the Earth (6378.135 km.)

(r, λ, δ) are respectively the geocentric distance, latitude and east longitude of the sub vehicle point from Greenwich,

$P_n^m(\sin\delta)$ are the associated Legendre polynomial of the first kind, of degree n, of order m and m $\leq$ n.

C_{nm} and S_{nm} are harmonic coefficients

Then the potential V (in this study) will be described as

$$V = \frac{A_2}{2r^3}(3\sin^2\delta - 1)$$

Where $A_2 = \mu R^2 J_2$

It is convenient to change the independent variable from the time (t) to the unperturbed true anomaly (f) by using the relation

$$\frac{df}{dt} = \left(\frac{a}{r}\right)^2 n\sqrt{1-e^2} \quad ,n = \sqrt{\frac{\mu}{a^3}}$$

So, the Lagrange planetary equation, will be

$$\left.\begin{aligned}
\left(\frac{da}{df}\right)_{ob} &= 2\left(\frac{r}{a}\right)^2 \frac{e\sin f}{n^2(1-e^2)}\left\{\frac{3\mu J_2}{r^4}R^2[3Sin^2 i\ Sin^2 u - 1]\right\} + \\
&\quad \frac{2}{n^2}\left(\frac{r}{a}\right)\left\{-\frac{3\mu J_2}{r^4}R^2 Sin^2 i\, sin(u)cos(u)\right\} \\
\left(\frac{de}{df}\right)_{ob} &= \left(\frac{r}{a}\right)^2 \frac{sin f}{n^2 a}\left\{\frac{3\mu J_2}{r^4}R^2[3Sin^2 i\ Sin^2 u - 1]\right\} + \left(\frac{r}{a}\right)^2 \frac{1}{n^2 ae}\left[\frac{P}{r} - \frac{r}{a}\right] * \\
&\quad \left\{-\frac{3\mu J_2}{r^4}R^2 Sin^2 i\, sin(u)cos(u)\right\} \\
\left(\frac{di}{df}\right)_{ob} &= \left(\frac{r}{a}\right)^3 \frac{cos(u)}{n^2 P}\left\{-\frac{3\mu J_2}{r^4}R^2 Sin\, i\, sin(u)\cos i\right\} \\
\left(\frac{d\Omega}{df}\right)_{ob} &= \left(\frac{r}{a}\right)^3 \frac{sin(u)}{n^2 P\, sin\, i}\left\{-\frac{3\mu J_2}{r^4}R^2 Sin\, i\, sin(u)\, cos\, i\right\} \\
\left(\frac{d\omega}{df}\right)_{ob} &= -\Omega' \cos i - \left(\frac{r}{a}\right)^2 \frac{cos f}{n^2 ae}\left\{\frac{3\mu J_2}{r^4}R^2[3Sin^2 i\ Sin^2 u - 1]\right\} + \\
&\quad \left(\frac{r}{a}\right)^2 \frac{sin f}{n^2 ae}\left(1 + \frac{r}{p}\right)\left\{-\frac{3\mu J_2}{r^4}R^2 Sin^2 i\, sin(u)cos(u)\right\} \\
\left(\frac{dM}{df}\right)_{ob} &= \left(\frac{r}{a}\right)^2 \frac{1}{\sqrt{1-e^2}} \\
&\quad -\left(\frac{r}{na}\right)^2 \frac{1}{a\sqrt{1-e^2}}\left[2\frac{r}{a} - \frac{(1-e^2)\cos f}{e}\right]\left\{\frac{3\mu J_2}{r^4}R^2[3Sin^2 i\ Sin^2 u - 1]\right\} \\
&\quad + \left(\frac{r}{an}\right)^2 \frac{\sqrt{1-e^2}\sin f}{ae}[P + r]\left\{-\frac{3\mu J_2}{r^4}R^2 Sin^2 i\, sin(u)cos(u)\right\}
\end{aligned}\right\} \quad (3)$$

3. Application and numerical results

3.1 The algorithm of problem:

Assume that the values for the orbital elements are given at the start time t = 0:

Step 1: Obtain the six orbital elements from TLE (**T**ow **L**ine **E**lement) for a constellation of Satellites.

Step 2: At t = 0 calculate the satellite position and velocity for the given orbital elements.

Step 3: At t = 0 compute the three components S, T, W of the perturbation force.

Step 4: At t = 0, using the given values for the orbital elements, and the radial, transverse, and lateral components of the perturbing accelerations, S, T, W compute the six rates of change of the orbital elements.

Step 5: Numerically integrate the rates of change over a time interval $\triangle t$ (by using a Runge-Kutta method).

Step 6: Determine the values of the orbital elements at the end of the time interval, go back to step 2, and repeat until the end time is reached.

3.2 Numerical Example

Numerical example for "Molniya" satellite constellation. (http://celestrak.com/molniya.txt)

Molniya 1-80					
Perigee (Km)		Apogee (Km)	Inclination (deg)	Period (min.)	Semi major axis(Km)
401		40,003	61.55	718.7	26577.518489
Two Line Element Set (TLE)					
1	21118U 91012A 13003.58929428 -.00001434 00000-0 55725-3 0 2195				
2	21118 061.5508 076.6737 7449661 272.4937 345.7705 2.00342991160342				
Molniya 1-81					
Perigee (Km)		Apogee (Km)	Inclination (deg)	Period (min.)	Semi major axis(Km)
672		39,681	62.3005	717.8	26552.454414
Two Line Element Set (TLE)					
1	21426U 91043A 13003.60264829 -.00000087 00000-0 -24908-2 0 8199				
2	21426 062.3005 057.5927 7345172 277.9569 313.0757 2.00633155157905				
Molniya 1-87					
Perigee (Km)		Apogee (Km)	Inclination (deg)	Period (min.)	Semi major axis(Km)
789		39,619	61.664	718.8	26574.63125
Two Line Element Set (TLE)					
1	22949U 93079A 13003.51843969 -.00000003 00000-0 00000+0 0 24				
2	22949 61.6639 221.3050 7303846 260.2914 18.0963 2.00316052134455				

Since the adopted physical constants are

R= 6378.135 Km.

μ = 398600.8 Km^3/sec^2

$J_2 = 1.08263\times10^{-3}$

Using the above values to compute the change in the six orbital elements for Molniya three satellite constellations, but in this work one satellite from this constellation will be mentioned.

Comparison between Observing and Model data for MOLNIYA 1-80

Orbital Element	At 3 Jan.2013 (Initial values)	(TLE) after 100 revolutions	MODEL	The variation	Error
a	26577.5184	26588.320	26577.524	5.6×10^{-3}	4.06×10^{-4}
e	0.7449661	0.7465474	0.7450589	9.2×10^{-5}	1.99×10^{-3}
i	61.5508	61.5602	61.551	2×10^{-4}	1.49×10^{-4}
Ω	76.6737	68.2282	72.674	3.99	-6.5×10-3
ω	272.4937	273.8575	273.139	3.5×10-1	2.62×10-3
f	206.332	240.145	206.85	5.18×10^{-1}	13.8×10^{-2}

The figures from (1 to 6) represent the relation between time (day) and a, e, *i*, Ω, ω, *f* respectively for satellite (1-80).

We note that the semi major axis increasing with the time after hundred revolutions as shown in Fig.(1) Also for argument of perigee and true anomaly we noted clearly that its increasing with time as shown in Fig.(5) and (6). But for eccentricity and inclination the changes nearly constant as shown in Fig. (2) and (3), and for Longitude of Ascending node the trend line show decreases for this element with time, as Shown in Fig. (4).

These features denoted that the oblateness force has great affect on the Longitude of Ascending node and the True Anomaly but for semi major axis, eccentricity and inclination has less effect.

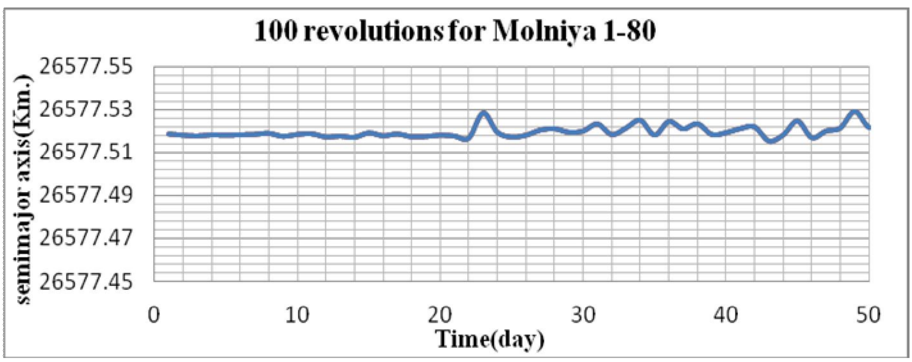


Fig.(1): Change of semi major axis with time for Molniya 1-80

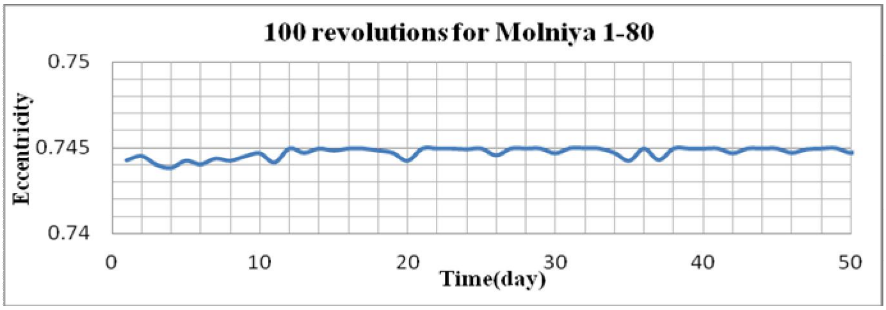


Fig. (2): Change of Eccentricity with time for Molniya 1-80

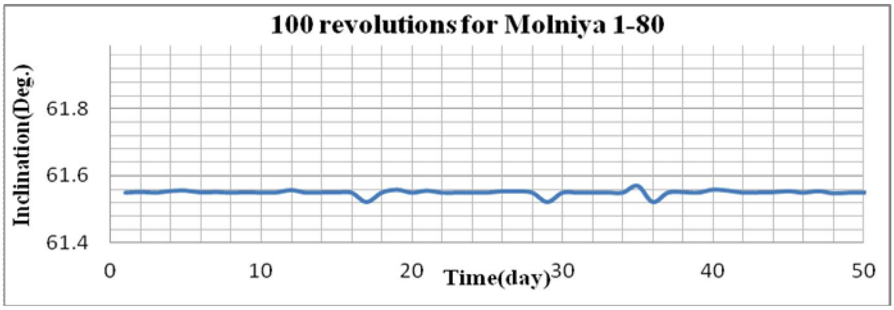


Fig.(3): Change of Inclination with time for Molniya 1-80

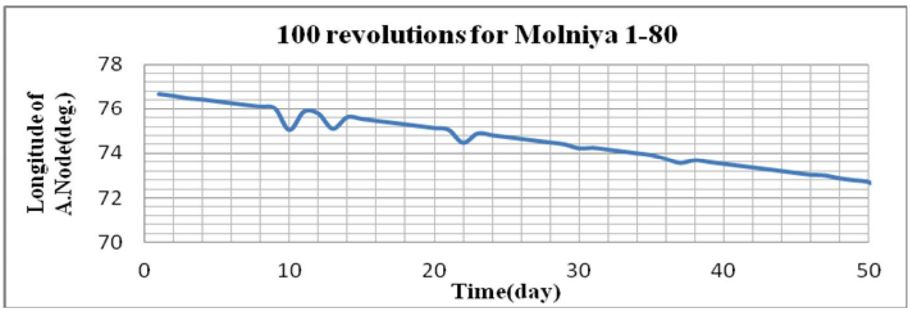


Fig.(4): Change of Longitude of A. Node with time for Molniya 1-80

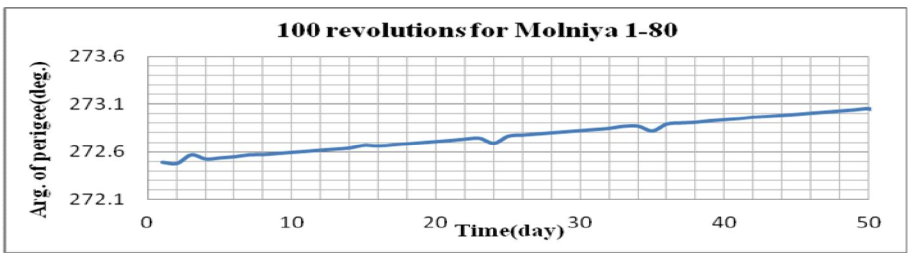


Fig.(5): Change of Arg. of perigee with time for Molniya 1-80

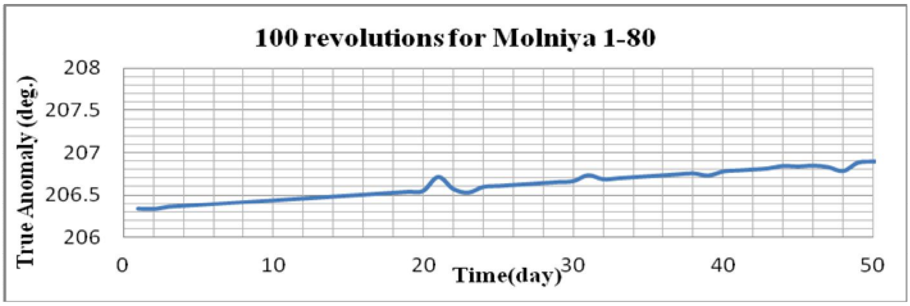


Fig.(6): Change of True Anomaly with time for Molniya 1-80

4. Conclusion

In this work, the definition of the reference orbit and via the simulative calculations are described; From the results its clear that the Rung-Kotta method is more convenient to compute the perturbations on the six orbital elements due to effect of oblateness force, because the results were nearly congruent with the observing data. So this model is more convenient to detect the perturbations on the orbital element due to J_2 and predict the position of any satellite in its orbit after interval time.

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