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Degree of Order Criteria of the Elements' Deterministic Chains With Relations Between the Closest Neighbors

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Abstract

For the first time ever, this work formulates definition of the degree of order for the non-random sequence of elements. Earlier, the only known parameter to evaluate a system's degree of order was entropy. It has been shown that using entropy to evaluate the degree of order in deterministic chains of elements leads to a contradiction. Words built out of k different elements are being considered. Compositions are built from a random number of closed words that in total contain an equal quantity of each of the k elements. Hereby a new approach is being offered for the degree of order definition based upon comparison of the given composition with a high-symmetry word composition. Composition is described by means of the matrix of the number of pairs, in which the sum of elements in each line and each column are equal. Features of the pair matrix are studied. Pair matrices are expanded into the matrices of ideal states that describe compositions of words with shift symmetry. By means of this expansion the degree of order had been defined and its combinatory meaning had been revealed. A model of the "cybernetic bug", able to cut words in certain places, had been studied. It had been shown that repeated attacks by this cybernetic bug together with subsequent random patching of the cut words may, in a relatively short period of time, transfer composition to the ideal state.

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1 Introduction. Entropy and Order

1.1 The background

Problems of entropy, order and amount of information have a long history, going back to Erwin Shroedinger's course of lectures (1943), later published as a monograph [1]. Advent of the Shannon's information theory [2, 3] gave new momentum to discussions of these problems, in particular in relation with the research of biological systems [4]. Analogy between entropy in thermodynamics and statistical physics on the one hand and amount of information of the other seemed very much attractive. The reality is, however, that notwithstanding single mathematical apparatus, these notions are quite different. Brillouin [5] was the first to note this. Entropy in statistical physics is defined through the number of microstates W, used to define a system's macrostate (”bound information”, according to Brillouin):

$$S = k \ln W, \tag{1.1}$$

where k is Boltzmann constant. Let us note that the microstates are real. A point, describing the system's microstate in the phase field, given sufficient time will pass arbitrary close from any possible state. The information theory has neither physical system nor macro- or microstates. The event space is purely conceptual. Probabilities may be set *a priori* or evaluated statistically. In this case Brillouin [5] is talking about the free information. In order to calculate information, contained in a book's text or in the DNA, one has to imagine an ensemble of such texts. In reality, however, no such ensemble exits. In thermodynamics lower level of entropy means higher degree of order. The perfect example is transfer from liquid into crystalline state, when entropy experiences a stepwise decrease. As a matter of fact, in this case entropy is the parameter of order, though the notion of the degree of order here is only accepted in a completely intuitive way.

1.2 Contradictions between entropy and order

However, can we generally assume that the degree of order within the system is described by entropy and that the degree of order is nothing but a steadily decreasing function of entropy? In case of deterministic systems such assumption leads to a paradoxical conclusion. Let us examine a long deterministic sequence, containing equal number of ones and zeroes. Calculating statistical conditional probabilities for the second order Markov chain, it is pretty easy to calculate information the above mentioned sequence contains [6, 7]. In case the sequence in question is ideally periodic with the period 01, then the information obtained will be equal $H = 1$ (bit), which corresponds to the information on the first symbol. If, somewhere in just one place, the sequence is broken (we have 0110 instead of 0101) , then $H = 2\log(n)$ (bit) , where n is the number of zeroes in the chain. Here we can see that the smallest breakdown in periodicity leads to a great at $n \gg 1$ leap in entropy. This may seem strange, though it goes with the theory of information and is identical to the infinite information of a black dot on the white background. Something else is interesting. In the above example entropy, at the minimal violation of order, logarithmically increases with the chain's length. If we presume that the degree of order is a steadily decreasing function of entropy, then in this case, with the periodic chain's elongation and preservation of defect in one place, the order should have been decreasing, which is absolutely inconsistent with intuitive perception of it, for it seems that with the chain elongation a single defect's specific weight decreases. All above said only means that notion of the degree of order does not have any rigorous definition today.

Entropy in physics is a measure of chaos in homogeneous systems. In theory of information entropy is a measure of uncertainty in a message and information is understood as eliminated uncertainty. Even from this notion it is clear that in the deterministic systems entropy is not a parameter to describe the degree of order.

Do we actually understand what order in biological systems is? There's one more example of paradoxical conclusion that emerges when using entropy. Bluemenfeld [4] shows that, according to thermodynamic criteria, any biological system is ordered not more than a piece of rock of the same weight. Evaluation is based on calculating a logarithm of a number of ways that may be used to build ordered structures an organism has: cells, proteins, DNA etc. The logarithm thus received is multiplied by the Boltzmann constant. The result is perplexing. It appears that, from the theory of information point of view, the living systems have only negligible amount of information. In fact, the problem is that degree of order has no clear definition.

1.3 The objective

It is generally accepted that any system of elements is highly ordered, provided it has symmetry groups. Now let us assume we are dealing with a deterministic system that has no symmetry, but there is a certain procedure to arrange it, i.e. transfer the system to the state where such symmetry exits. In the latter case one may formulate a very general question. How close such a system would be to the state of symmetry? The study of this question in relation to the sequence composed of equal number of included elements is the major objective of this work. The discovered arrangement algorithm has a very specific combinatorial meaning allowing evaluation of the sequence of elements' degree of order. In future, definition of the degree of order of more complex systems is of great interest.

2 Description of Composition of Closed Words by Means of the Pair Matrix

Let there be k different elements. These elements may be of any nature, say molecules, symbols etc. etc. Let each element have two connections - left and right. In this way elements can create linear chains or, as the theory of information calls them, "words". The chains may be open or closed into a ring. In particular, if only one element exists, we assume that it also can close its connections to form a ring of its own. We intend to build a function that describes a chain's degree of order. As an order template we choose a certain set of periodic chains. Our function shall show how much the given word differs from the template. In order to realize this idea, as we will see below, we would need to examine not a separate chain, but a set of closed words, which in aggregate has the equal number of all possible elements. Let us examine three chains:

$$112321232211133311212232323 3 \rightarrow \qquad (a)$$

$$123123123123123123123123123 \rightarrow \qquad (b)$$

$$111111111232323232323232323 \rightarrow \qquad (c)$$

The digits here represent three different elements, arrow on the right means that the chain is a circle, i.e. the last element is connected to the first one. Based on intuition, chain (a) should be referred to as pretty much unordered, chain (b) - a chain of perfect order, because it is periodic and, as a result, highly symmetric - it coincides with itself if shifted by 3 elements. What concerns the third example, then the sequence is very close to the notion of the "ideal order", but it is not symmetric relative to any shift. However, sequence (c) can produce two highly symmetric fragments with one step of the following procedure. Let us cut in (c) two connections 3-1 and 1-2 and close the resulting fragments into rings:

$$111111111 \rightarrow \qquad (d)$$

$$232323232323232323 \rightarrow \qquad (e)$$

Just one step of the above procedure resulted in two words of high symmetry. It is perfectly clear that the similar procedure for the word (a) will be much longer. These simple considerations give birth to the following approach to determine the degree of order.

So, let there be an initial set, in which each of k elements takes place exactly n times ($k \geq 2, n \geq 1$). Let us create a set of s closed words (fragments), in which all nk of the elements are used. Let us call this set a composition. Let us emphasize that each element in the composition is inevitably connected to the neighboring elements to its right and left or closed to itself. Let us give an example for $k = 4,\ n = 5,\ s = 3$:

$$1241 \rightarrow \qquad 13332 \rightarrow \qquad 24444113322 \rightarrow$$

Let us designate elements of the alphabet, the composition had been built from, by indices i, j:

$$1 \leq i \leq k, \qquad 1 \leq j \leq k.$$

Sequence of any two consecutive left to right elements in the word we'll call a pair, for brevity sake. Let us designate the total number of pairs in the words of composition that have element i to the left and element j to the right as a_{ij}. (Let us note that every element of the composition is found in two pairs and, in general case, $a_{ij} \neq a_{ji}$). Therefore, in order to describe the composition we obtain a matrix of the number of pairs:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & \dots & \dots & \dots \\ a_{k1} & a_{k2} & \dots & a_{kk} \end{pmatrix}. \tag{2.1}$$

As soon as all words in the composition are closed in a ring, for $1 \leq i \leq k$ we have

$$\sum_{j=1}^{k} a_{ij} = \sum_{j=1}^{k} a_{ji} = n. \tag{2.2}$$

It is understood, that different compositions, containing different number of fragments, can correspond to the pair matrix (2.1).

Among all possible compositions let us discriminate the "ideal order compositions" that have the following structure:

- All composition fragments are periodic, i.e. they possess shift symmetry.
- Each of the k elements can belong to only one period, but it may also belong to different fragments.
- In any fragment the smallest period does not contain any repeated elements.

For example, a pair of above mentioned words $(d), (e)$ produces an ideal order composition, where $k = 3,\ n = 9,\ s = 2$. In order to determine the number of different ideal order compositions, calculation of a number of different compositions built of k different elements ($n = 1$), will be sufficient. In total, we get $k!$ different ideal order compositions; at that, with any n each ideal composition is invariably represented by the distinct set of periods.

Let us examine one of the ideal order compositions that has $s = k$ fragments, each having n identical elements, at that. Such composition corresponds to complete and orderly separation of all elements. The pair matrix for such composition will be $U_1 = nI^k$, where I^k- is a unit matrix of order k. As soon as a particular element is also a closed word, matrix U_1 describes all compositions, in which only identical elements connect with each other, at that $k \leq s \leq nk$. Pair matrices for other ideal order compositions result from rearranging U_1 lines. If we designate s_{min} and s_{max} as minimal and maximal possible number of fragments for the ideal composition correspondingly, then $s_{max} = ns_{min}$.

Let us exemplify matrices of ideal compositions for $k = 2$ and $k = 3$. In case of $k = 2$ we have:

$$\mathbf{U_1} = n \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mathbf{U_2} = n \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.3}$$

In case of minimal number of fragments U_1 corresponds to two words: $111...1 \rightarrow$ and $000...0 \rightarrow$, whereas U_2 describes chain $0101...01 \rightarrow$. In case of $k = 3$ we have 6 ideal compositions, described by matrices:

$$\mathbf{U_1} = n \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{U_2} = n \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{U_3} = n \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$
$$\mathbf{U_4} = n \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{U_5} = n \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad \mathbf{U_6} = n \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{2.4}$$

As it has been mentioned earlier, there is no bijective relation between composition of words and pair matrix. Nevertheless, the pair matrix carries sufficient information on the said composition's proximity to the state of ideal order. To see this, let us examine the pair matrix' expansion to matrices of ideal compositions.

3 Expansion of Pair Matrix to Ideal Order Matrices

First, let us examine the vector space of square matrices M_k of k order with real components $a_{ij} \in R$. Let $V \subset M_k$ represent the linear subspace, defined in the following way:

$$V = \{A \in M_k | \quad \forall \quad 2 \leq i,j \leq k, \quad \sum_{s=1}^{k} a_{sj} = \sum_{s=1}^{k} a_{is} = \sum_{s=1}^{k} a_{1s} \neq 0\}. \tag{3.1}$$

In (3.1) we have $2k-2$ linear independent homogeneous conditions, superimposed on the M_k matrix' components. U_i matrices comply with the same conditions. Let us examine U set of all ideal order matrices.

$$U = \{U_1, U_2, ..., U_{k!}\}. \tag{3.2}$$

It is easy to show, that among $k!$ ideal order matrices one can always select $(k-1)^2 + 1$ linearly independent matrices that form a basis set of subspace V. Then it becomes obvious that:

$$V = span(U). \tag{3.3}$$

Dimension of subspace V equals:

$$dimV = k^2 - 2k + 2. \tag{3.4}$$

Set of pair matrices, describing compositions with equal number of all possible elements, is a subset of the linear subspace V, in which a_{ij} are nonnegative integers. Though such a subset is not a linear subspace, inside it expansions can be built similar to expansion in basis set in the linear subspace V.

Theorem 1. *Any pair matrix $A = \{a_{ij}\}$ can be expanded to a combination of r matrices of ideal compositions, in which the expansion coefficients x_i are natural numbers:*

$$A = \frac{1}{n} \sum_{i=1}^{r} x_i U_{0i}, \quad U_{0i} \in U. \tag{3.5}$$

Expansion features:

- Expansion(3.5) is many-valued in general way.
- There is an optimal expansion, in which the number of ideal order matrices in (3.5) does not exceed dimension of subspace V: $r \leq (k-1)^2 + 1$.
- Generally speaking, the set of matrices U_{0i} in (3.5) is different for different A.

Proof. First, let us examine expansion of matrix A with real components, arising from (3.3):

$$A = \frac{1}{n} \sum_{i=1}^{k!} x_i U_i, \quad A \in V, \quad x_i \in R. \tag{3.6}$$

The system of equations (3.6) for x_i contains k^2 equations. Pair matrices A, U_i comply with $2k-2$ linear independent conditions (3.1). Therefore, system (3.6) is compatible and contains $k^2 - (2k-2)$ independent equations and l slack variables x_i :

$$l = k! - (k^2 - 2k + 2). \tag{3.7}$$

At $k = 2$ in (3.7) we have $l = 0$ and, consequently, the system (3.6) has the single unique solution. In case of $k \geq 3$, the system (3.6) has infinitely many solutions. The aforesaid is also true for pair matrices with nonnegative integer components, discussed further. In the later case we are interested only in natural solutions x_i complying with equation (3.6). Further on we'll make sure that such solutions always exist.

Let us build an expansion algorithm for matrix A with nonnegative integer components to matrices of symmetrical states. In matrix A let us randomly select k nonzero elements, located in different lines and columns. Let us assume the smallest of them equals x_1. Let us present matrix A in the following form:

$$A = A_1 + \frac{x_1}{n} U_{01},$$

where U_{01} is an ideal composition matrix, in which nonzero components are placed in correspondence to the selected above elements of A; A_1 is a residual matrix with pair matrix properties and having, in comparison to A, minimum one extra zero element. Let us repeat the same procedure with matrix A_1, labeling the smallest of the selected elements x_2, a new residual matrix A_2, and so forth. We get a recurrent procedure:

$$A_{i-1} = A_i + \frac{x_i}{n} U_{0i}, \quad i \geq 1, \quad A_0 = A. \tag{3.8}$$

The procedure (3.8) will end as soon as the residual matrix is equal to zero. We get expansion A as (3.5), where all $x_i > 0$, all U_{0i} are different and, therefore, $r \leq k!$. It is obvious that at $k > 2$ expansion (3.5) is many-valued. Now let us show that in reality the latter procedure results in optimal expansion, in which $r \leq k^2 - 2k + 2$. After i steps of the recurrent procedure, matrix A_i contains $i_1 \geq i$ zeroes. In this case, $2k - 2 + i_1$ linear homogenous conditions are being superimposed on k^2 elements of matrix A_i ($2k-2$ independent homogenous conditions in (3.1) plus i_1 zeroes, received in A_i). As soon as the process of iterations shows that the number of all matrix A_i elements equals the number of linear independent homogenous conditions, superimposed on a_{ij} , that is when

$$k^2 = 2k - 2 + i_1, \tag{3.9}$$

we obtain that $a_{ij} = 0$, in other words, maximum for $i_{max} = k^2 - 2k + 2$ iterations pair matrix A_i zeroes out. Thus, for r in (3.5) we have: $r \leq k^2 - 2k + 2$ and the theorem is proved. Let us note, that if for some i of the recurrent procedure (3.8) a natural x_i is selected smaller than the smallest element in transverse of A with nonzero elements in U_{0i} , then in (3.5) we may get a non-optimal expansion, in which $k^2 - 2k + 2 < r \leq k!$.

4 Composition Degree of Order

If we superimpose matrix U_i on matrix A and point out the smallest element g_i in the transverse of nonzero elements in U_i with elements A, we'll find out that in (3.5) coefficient x_i before U_i complies with the following inequation:

$$0 \le x_i \le g_i. \tag{4.1}$$

In this way, among all expansions (3.5) of matrix A, in which U_i is being found, the maximum possible coefficient before U_i equals g_i. Now let us determine parameter G, which equals the maximum possible coefficient x_i in expansion (3.5).

$$G = max(g_i), \qquad 1 \le i \le k!. \tag{4.2}$$

Value G , determined in (4.2) shows how close the present composition is to the state of ideal symmetry. In this sense G determines composition's degree of order.

Now let us examine the extreme features of G parameter at the fixed n. First of all, if composition of words in question is in one of the ideal symmetry states, we have $G = G_{max} = n$. Further let us review the states that may be called chaotic and discuss minimal value of G.

4.1 Equiprobable distribution of pairs

Let n be aliquot to k. Then an absolutely homogenous state of composition is possible, where all matrix elements are equal: $a_{ik} = n/k$. This state is of interest because, as the classic theory of information goes, in homogenous state entropy is maximal. In this case a pair matrix may be expanded to k matrices of ideal state with equal coefficients $x_i = G = n/k$. (This is because among the ideal order matrices there are k matrices, in which all ones are located in different places). Now, let residue of dividing n by k equal p. Then the closest to homogenous state of composition is possible, in which $k - p$ elements in the pair matrix line equal $[n/k]$, i.e. the integer part of n/k, whereas the rest of elements equal $[n/k] + 1$. Then $G = [n/k] + 1$. In this way, at the closest to homogenous state composition, we get:

$$G = \begin{cases} n/k, & p = 0 \\ [n/k] + 1, & p \ne 0 \end{cases} . \tag{4.3}$$

4.2 Minimum G at $k = 2$

In this case expansion of the composition to U_1, U_2 is unique: $A = x_1U_1 + x_2U_2$. We get minimum G at the fixed n if $|x_1 - x_2| \le 1$, which complies with formula (4.3) at $k = 2$. At $k > 2$ minimum G does not comply with the homogenous distribution of pairs anymore.

4.3 Minimum G at $k = 3$

Let n be aliquot to 4 . Let us examine a composition, pair matrix of which is expanded to any 4 matrices of symmetric states with expansion coefficients of $x_i = n/4$. In this case the pair matrix contains one zero element, expands uniquely and takes, for example, the following form:

$$\mathbf{A} = \frac{n}{4} \begin{pmatrix} 2 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 2 & 2 \end{pmatrix}, \tag{4.4}$$

From (4.4) it follows that $G = n/4$. Now, let assume that residue of dividing n by 4 equals p. In order to get minimum G in this case, let us examine a pair matrix, built out of the same symmetric state matrices, as in (4.4). At that, we'll select p expansion coefficients equal to $x_i = [n/4] + 1$, the rest will be $x_i = [n/4]$. In this way we get:

$$G = \begin{cases} n/4, & p = 0 \\ [n/4] + 1, & p \neq 0 \end{cases} . \quad (4.5)$$

If we examine the similar expansion to $5\ U_i$ matrices, we'll get $G = 2n/5$ or $G = 2[n/5] + 1$, which is more than in (4.5). (Uniform expansion to $6\ U_i$ matrices is similar to equiprobable distribution of pairs and has been discarded above). Therefore, in (4.5) we have minimum G at $k = 3$ and fixed n. We assume that this minimum corresponds to higher degree of derangement at $n \geq 4$, then when all elements of the pair matrix are equal. For side-by-side comparison, let us examine two compositions of equal length $(n = 12)$, below . Each of them represents one word. The first one is made of three, whereas the second one - of four similar areas:

$$211121323332 \to \times 3, \qquad \mathbf{A} = \begin{pmatrix} 6 & 3 & 3 \\ 6 & 3 & 3 \\ 0 & 6 & 6 \end{pmatrix}, \qquad G = 3; \quad (4.6)$$

$$332211231 \to \times 4, \qquad \mathbf{A} = \begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \end{pmatrix}, \qquad G = 4. \quad (4.7)$$

In view of the above we have to assume that word (4.7) is more symmetric, than (4.6). At first sight this must seem natural, not least because the period in (4.7) is shorter than the one in (4.6). However, to matrices A and B in (4.6, 4.7) may correspond completely non-periodic words, as well as compositions built of several fragments. However, the combinatorial meaning of parameters G and g_i , examined below, actually justifies awarding degree of order sense to parameter G.

4.4 Minimum G at $k = 4$

Let $n \geq 6$ .Then we may assert that G_{min} corresponds to almost uniform expansion of A to six ideal order matrices of. If n is aliquot to 6, then matrix, corresponding to G_{min}, is similar to (4.4) and takes, in particular, the following form:

$$A = \frac{n}{6} D, \qquad \mathbf{D} = \begin{pmatrix} 3 & 1 & 2 & 0 \\ 1 & 3 & 2 & 0 \\ 1 & 1 & 1 & 3 \\ 1 & 1 & 1 & 3 \end{pmatrix}. \quad (4.8)$$

(A sufficient condition of matrix A in this case to expand to maximum possible number of U_i matrices having equal coefficients x_i is presence in matrix D of two identical lines with all but one elements equal 1). Expansion of matrix (4.8) is many-valued. It is clear, however, that $G = n/6$. At an arbitrary $n \geq 6$ for G_{min} we have the result identical to (4.5), in which $n/4$ should be replaced by $n/6$. For an arbitrary k the formula for G_{min} is not found yet.

5 Composition Regulation Procedure

Let a composition be given. Let us introduce the following arrangement procedure. In the composition words we are allowed to make exactly k cuts, subsequently patching the fragments thus obtained to build a new composition. Ends of a separate fragment can also be patched together. Let us call each procedure as above an arrangement step. Two questions arise. What is the minimum number of steps required to rearrange present composition into the given ideal state or into the closest (by number of steps) ideal state? The answers are found in parameters g_i and G.

Let us cut the composition k times in such a way that each element, found at the ends of the cut fragments, would appear exactly twice - once on the left and once on the right. This is always possible. Indeed, if during superimposition of some matrix U_m on the pair matrix A, nonzero elements U_m fail to superimpose on zero in A, then we need to cut those pairs that correspond to nonzero elements U_m. (It is easily seen that at least one such U_m matrix always exists.) Let us call the described procedure a U_m-cut. Now, the cut open fragments should be patched back. Inasmuch as each element is found twice - once with the left and once with the right connection - it is always possible to get a new composition, the pair matrix of which will be A':

$$A' = A + \frac{1}{n}(-U_m + U_i), \tag{5.1}$$

where U_i is the matrix of the ideal composition we want to transfer the original one into. Hence, the minimum number of steps required to transfer the original composition into U_i state, equals:

$$N_i = n - g_i. \tag{5.2}$$

The minimum number of steps required to transfer to the closest ideal state may serve the degree of order criterion:

$$N = n - G. \tag{5.3}$$

The introduced arrangement procedure leaves an open question as to the number of composition fragments after each step, as well as the final symmetric state. In particular, if the original composition was a single closed word, it would have been interesting to establish the minimum possible number of fragments in the given final ideal state. Let us consider an example. Let a one word composition $(k = 4, n \geq 2)$ be given:

$$11 \cdots 122 \cdots 233 \cdots 344 \cdots 4 \rightarrow \tag{5.4}$$

By cutting connections $12, 23, 34, 41$ and closing each cut fragment to itself, in one step we transfer the given composition (5.4) into the state of complete partition of elements with $s_{min} = 4$ fragments, therefore $G = n - 1$, $N = 1$. Now let us examine the number of transfers of composition (5.4) into another ideal state, described by matrix

$$\mathbf{U_i} = n \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}, \tag{5.5}$$

which the following one word composition $(s_{min} = 1)$ corresponds to:

$$12341234...1234 \rightarrow .$$

In this case, for the matrix corresponding to (5.4), we have $g_i = 1$, $N = n-1$. If n is odd, it is possible to execute U_m-cuts in (5.4) in such a manner that the final state U_i would be represented by a single fragment. If n is even, the minimum number of fragments in U_i state equals 2. In general, studying different examples, we can find words arrangeable in such a way that the number of fragments in the ideal state would be s_{min}.

6 Random Self-arrangement and Cybernetic Bug

Let us assume that a set of kn elements can create random compositions with random sequence of elements. Let us further assume that open-end fragments can be patched to each other (or to itself) in a random way. Let there be a cybernetic bug that attacks this composition and during one U_m-cut take place per attack. At that, the bug is able of making any of the U_m-cuts, except one, say U_1 . The attack completed, the bug retires for some time, and during this time the cut fragments patch together in a random way. Let there be L attacks. The question arises - how many attacks are required to transfer the composition into U_1 state with probability of $1-\gamma$ ($\gamma \ll 1$)? When patching, the cut fragments could be patched together in $k!$ ways. Let all ways be equally probable. Probability of $U_i = U_1$ in the new composition (5.1) is

$$p = \frac{1}{k!}. \tag{6.1}$$

That is to say, following the cybernetic bug's attack the composition will be one step closer to the ideal order U_1 with probability p. Let the number of steps required for transition to the ideal state U_1 equal $N \gg 1$. Let us assume that $L > N$. Probability P_L of the composition's transition to the ideal state after L attacks is:

$$P_L = 1 - \sum_{i=0}^{N-1} \binom{L}{i} p^i (1-p)^{L-i}. \tag{6.2}$$

We would like to choose L in such a way that

$$1 - P_L \lesssim \gamma. \tag{6.3}$$

Let us confine ourselves to a sufficiently small γ, so that solution of inequality (6.3) would be in the area of $L > 2N$. Then we may use the following evaluation:

$$\binom{L}{i} < \binom{L}{N} < \frac{L^N}{N!}. \tag{6.4}$$

Condition (6.3) would be met *a fortiori*, if

$$\frac{L^N}{N!}(1-p)^L u = \gamma, \tag{6.5}$$

$$u = \begin{cases} N, & p = 0.5 \quad (k=2) \\ \frac{1-p}{1-2p}, & p > 0.5 \quad (k>2) \end{cases}.$$

To evaluate L in (6.5) it would be enough to evaluate the larger root of the equation:

$$L - a\ln L - b = 0, \tag{6.6}$$

$$a = -\frac{N}{\ln(1-p)}, \qquad b = \frac{1}{\ln(1-p)} \ln(\frac{\gamma N!}{u}).$$

The graph of the function $y(L) = L - a\ln L - b$ is concaved with its minimum at $L = a$:

$$y_{min} = y(a) = a - a\ln a - b.$$

Given sufficiently large N and small γ, complying with inequality

$$\frac{\gamma}{u}\sqrt{2\pi N}[\ln\frac{1}{1-p}]^N < 1,$$

we get $y_{min} < 0$. This permit to use $L = a\ln a + b > a$ as the first approximation of equation (6.6) solution. Further, using the Newton method, we get evaluation:

$$L \gtrsim (a\ln a + b)\left(1 + \frac{\ln(\ln a + \frac{b}{a})}{\ln a + \frac{b}{a} - 1}\right). \tag{6.7}$$

For comparison let us examine a purely chaotic way of reaching the ideal order, when kn elements randomly join into an open chain. If the required ideal order is not reached, the chain is destroyed and the experiment repeats itself. If the order is reached, the experiment stops. Let us consider all test results equiprobable with probability of

$$p = \left\{ \frac{(kn)!}{(n!)^k} \right\}^{-1}.$$

Let us discuss the following question - what minimum number of tests L is to be done to complete the experiment with probability of $1-\gamma$. Probability of the experiment to complete after the number of tests not exceeding L is

$$P_L = p \sum_{i=1}^{L} (1-p)^{i-1}. \tag{6.8}$$

Hence, from the requirement $1 - P_L = \gamma$ at $n \gg 1$ we get:

$$L = \frac{(kn)! \ln(\gamma^{-1})}{(n!)^k}. \tag{6.9}$$

Let us compare (6.9) with the result of (6.7), where $p = 1/k!$. Let $k = 3$, $\gamma = 0.02, \quad n = 40$. At that, pending the cyber bug's attacks the sequence was in the state of chaos: $N \simeq n/4$. Then from (6.7) $L \approx 240$, and from (6.9) $L \approx 4.8 \cdot 10^{55}$. In this way, the cybernetic bug, destroying only specific connections in the composition, can, in a relatively small number of steps, bring it to the state of order, while purely random arrangement seems absolutely inconceivable.

7 Linear Function from the Pair Matrix Elements

Let us examine the next function:

$$F(A) = \sum_{i=1}^{k} \sum_{j=1}^{k} \varepsilon_{ij} a_{ij}, \qquad \varepsilon_{ij} \in \mathbf{R} \tag{7.1}$$

Function (7.1) is an additive one in relation to operation of compositions' merger. Beside operation of merger we may examine operation of fusion, when cutting and patching of fragments of the compositions being fused are allowed. If fusion is carried out in such a way that pair matrix of the resulting composition is equal to the sum of matrices of all compositions being fused, then function (7.1) would also be additive. A physical analogy for function (7.1) is bonding energy in the molecular chain in the closest neighbors' model. Let us express $F(A)$ through $F(U_l)$. In accordance with (3.5) we get:

$$F(A) = \sum_{i=1}^{k} \sum_{j=1}^{k} \varepsilon_{ij} \frac{1}{n} \sum_{l=1}^{r} x_l (U_l)_{ij} = \sum_{l=1}^{r} x_l \varepsilon_l, \tag{7.2}$$

$$\varepsilon_l = \sum_{i=1}^{k} \sum_{j=1}^{k} \varepsilon_{ij} \frac{1}{n} (U_l)_{ij},$$

$$0 \le x_l \le n, \qquad \sum_{l=1}^{r} x_l = n.$$

Let us assume that with given ε_{ij} the maximum value of ε_l reaches at a single $l = l_1$, whereas its minimum value reaches only at $l = l_2 \neq l_1$. In this case we'll call U_{l_1} and U_{l_2} states the order poles. From (7.2) we obtain next theorem.

Theorem 2. *Suppose that order poles exist. Then linear function (7.1) reaches its maximum and minimum if and only if the composition is in U_{l_1} and U_{l_2} states respectively, at that:*

$$F_{max} = n\varepsilon_{l_1}, \qquad F_{min} = n\varepsilon_{l_2}.$$

It is understood that an order pole degenerates if and when several values of l correspond to the extreme value of ε_l.

8 Conclusion

According to the research conducted above, definition of the degree of order cannot be universal. Degree of order can only be defined as a function of a specific arrangement procedure. In particular, the Kolmogorov complexity [8, 9] can be regarded as a criterion of order, in which the arrangement procedure is the shortest algorithm by means of which a given word can be reproduced. Within the discussed set and introduced arrangement procedure, the following questions remain open:

- General formula for the degree of order minimum at the given word length.
- Number of fragments in the arrangement procedure.

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Competing Interests

The author declares that no competing interests exist.

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