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A Self-Starting Five-Step Eight-Order Block Method for Stiff Ordinary Differential Equations

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

This paper examines the implementation of a self-starting five-step eight-order block method with two off-grid for stiff ordinary differential equations using interpolation and collocation procedures. The predictor schemes are then expanded using Taylor's series expansion. Multiple numerical integrators were produce and arrived at a discrete scheme. The discrete schemes are of uniform order eight and are assembled into a single block matrix equation. These equations are simultaneously applied to provide the approximate solution for stiff initial value problem for ordinary differential equations. The order of accuracy and stability of the block method is discussed and its accuracy is established numerically.

Keywords: Block method; stiff; five-step; power series.

1 Introduction

A considerable literature exist for the conventional k-step linear multi-step methods for the solution of ordinary differential equations (ODE's) of the form

$$y' = f(x, y)\ ,\ \ y(x_0) = y_0\ ,\ \ x \in [a, b] \tag{1}$$

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The solution is in the range $a \le x \le b$, where a and b are finite and we assumed that f satisfies the Lipschitz condition, which guarantees the existence and uniqueness of the solution of the problem (1).

For the discrete solution of (1) by linear multi-step method has being studied by authors like [1] and continuous solution of (1) [2] and [3,4]. One important advantage of the continuous over discrete approach is the ability to provide discrete schemes for simultaneous integration. These discrete schemes can be reformulated as general linear methods (GLM) [5]. The block methods are self-starting and can be applied to both stiff and non-stiff initial value problem in differential equations.

Definition 1. *A stiff equation is a differential equations that are characterized as those whose exact solutions has a term of the form* e^{-ct} *, where* c *is a large positive constant.*

More recently, authors like [6,7,8,9] and [10] to mention few, these authors proposed methods ranging from predictor- corrector to hybrid block method for initial value problem in ordinary differential equation.

In this work, we derived the continuous self-starting five-step order-eight block method with two off-grid using Taylor's series expansion. This would help in coming up with a more computationally reliable integrator that could solve stiff differential equations problems of the form (1).

2 Derivation Technique of the Self-starting Block Method

We consider the continuous hybrid formula to be the form

$$y(x) = \sum_{j=0}^{s+t-1} \alpha_j x^j \tag{2}$$

Interpolation and collocation procedures are use by choosing interpolation point t at a grid point and collocation points s at all points giving rise to $\xi = t + s$ system of equations whose coefficients are determined by using appropriate procedures. The first derivative of (2) is given by;

$$y' = \sum_{j=0}^{s+t-1} j\alpha_j x^{j-1} \tag{3}$$

Then, substituting (3) in (1), we obtain

$$f(x,y)' = \sum_{j=0}^{s+t-1} j\alpha_j x^{j-1} \tag{4}$$

Now, collocating (4) at point x_{n+s} , $s = 0(1)5$ and interpolating (2) at points $x_{n+\frac{3}{2}}$ and $x_{n+\frac{5}{2}}$ leads to the following system of equations;

$$AX = U \tag{5}$$

$$A = \begin{bmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \end{bmatrix}^T ,$$

$$U = \begin{bmatrix} y_n & f_n & f_{n+1} & f_{n+\frac{3}{2}} & f_{n+2} & f_{n+\frac{5}{2}} & f_{n+3} & f_{n+4} & f_{n+5} \end{bmatrix}^T ,$$

$$X=\begin{bmatrix} 1 & x_n & x_n^{2} & x_n^{3} & x_n^{4} & x_n^{5} & x_n^{6} & x_n^{7} & x_n^{8} \\ 0 & 1 & 2x_n & 3x_n^{2} & 4x_n^{3} & 5x_n^{4} & 6x_n^{5} & 7x_n^{6} & 8x_n^{7} \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^{2} & 4x_{n+1}^{3} & 5x_{n+1}^{4} & 6x_{n+1}^{5} & 7x_{n+1}^{6} & 8x_{n+1}^{7} \\ 0 & 1 & 2x_{n+\frac{3}{2}} & 3x_{n+\frac{3}{2}}^{2} & 4x_{n+\frac{3}{2}}^{3} & 5x_{n+\frac{3}{2}}^{4} & 6x_{n+\frac{3}{2}}^{5} & 7x_{n+\frac{3}{2}}^{6} & 8x_{n+\frac{3}{2}}^{7} \\ 0 & 1 & 2x_{n+2} & 3x_{n+2}^{2} & 4x_{n+2}^{3} & 5x_{n+2}^{4} & 6x_{n+2}^{5} & 7x_{n+2}^{6} & 8x_{n+2}^{7} \\ 0 & 1 & 2x_{n+\frac{5}{2}} & 3x_{n+\frac{5}{2}}^{2} & 4x_{n+\frac{5}{2}}^{3} & 5x_{n+\frac{5}{2}}^{4} & 6x_{n+\frac{5}{2}}^{5} & 7x_{n+\frac{5}{2}}^{6} & 8x_{n+\frac{5}{2}}^{7} \\ 0 & 1 & 2x_{n+3} & 3x_{n+3}^{2} & 4x_{n+3}^{3} & 5x_{n+3}^{4} & 6x_{n+3}^{5} & 7x_{n+3}^{6} & 8x_{n+3}^{7} \\ 0 & 1 & 2x_{n+4} & 3x_{n+4}^{2} & 4x_{n+4}^{3} & 5x_{n+4}^{4} & 6x_{n+4}^{5} & 7x_{n+4}^{6} & 8x_{n+4}^{7} \\ 0 & 1 & 2x_{n+5} & 3x_{n+5}^{2} & 4x_{n+5}^{3} & 5x_{n+5}^{4} & 6x_{n+5}^{5} & 7x_{n+5}^{6} & 8x_{n+5}^{7} \end{bmatrix}$$

Solving (5) , for $\alpha'_j\, s,\ j=0(1)8$ and substituting back into (2) gives a continuous linear multistep method of the form

$$y(x)=\alpha_0(x)\,y_n+h\sum_{j=0}^{7}\beta_j(x)\,f_{n+j}\,, \tag{6}$$

Where $\alpha_o=1$ and the coefficient of f_{n+j} gives

$$\beta_o=\frac{-1}{75600}x^5\begin{pmatrix}-20874h^3-24990x_nh-9576x_n^{2}h-1176x_n^{3}+4165xh^2+3192xx_nh\\ +588xx_n^{2}-456x^2h^{15}-168x^2h^{14}x_n+21x^3\end{pmatrix}/h^7$$

$$\beta_1=\frac{1}{30240}x^4\begin{pmatrix}420945h^5+824040x_nh^3+549150x_n^{2}h^2+151200x_n^{3}h+14700x_n^{4}-164808xh^3\\ 219660xx_nh^2-90720xx_n^{2}h-11760xx_n^{3}+36610x^2x_nh+5880x^2x_n^{2}-4320x^3h\\ -1680x^3x_n+210x^4\end{pmatrix}/h^7$$

$$\beta_2=\frac{-8}{945}x^5\begin{pmatrix}-2100h^3-2940x_nh^2-168x_n^{3}-1260x_n^{2}h+490xh^2+84xx_n^{2}+420xx_nh-24x^2x_n\\ -60x^2h+3x^3\end{pmatrix}/h^7$$

$$\beta_3=\frac{1}{1680}x^4\begin{pmatrix}99295h^4+219380x_nh^3+160650x_n^{2}h^2+47600x_n^{3}h+4900x_n^{3}-43876xh^{10}\\ -64260xh^9x_n-28560xh^8x_n^{2}-3920xh^7x_n^{3}+10710x^2h^2+9520x^2x_nh+1960x^2x_n^{2}\\ -1360x^3h-560x^3x+70x^4\end{pmatrix}/h^7$$

$$\beta_4=\frac{-8}{4725}x^4\begin{pmatrix}59220x_nh^3+45150x_n^{2}h^2+13860x_n^{3}h+1470x_n^{4}+25683h^4-11844xh^3-18060xx_nh^2\\ -8316xx_n^{2}h-1176xx_n^{3}+3010x^2h^2+2772x^2x_nh+588x^2x_n^{2}-396x^3h-168x^3x_n+21x^4\end{pmatrix}/h^7$$

$$\beta_5=\frac{1}{15120}x^4\begin{pmatrix}423150x_n^{2}h^2+134400x_n^{3}h+14700x_n^{4}+535920x_nh^3-225015h^4+169260xx_nh^2\\ -80640xx_n^{2}h-11760xx_n^{3}-107184xh^3+28210x^2h^2+26880x^2x_nh+5880x^2x_n^{2}\\ -3840x^3h-1680x^3x_n+210x^4\end{pmatrix}/h^7$$

$$\beta_6=\frac{-1}{30240}x^4\begin{pmatrix}25200x_n^{3}h+2940x_n^{4}+74550x_n^{2}h^2+89460x_nh^3+37191h^4-15120xx_n^{2}-2352xx_n^{3}\\ -29820xx_nh^2-17892xh^3+5040x^2x_nh+4970x^2h^2-720x^3h-336x^3x_n+42x^4\end{pmatrix}/h^7 \tag{7}$$

$$\beta_7=\frac{1}{8400}x^5\left(-56x_n^{3}-336x_n^{2}h-630x_nh^2-364h^3+28x_n^{2}x+112xx_nh+105xh^2-8x^2x_n-16x^2h+x^3\right)/h^7$$

Evaluating (7) at $x = 1(1)7$ gives a continuous discrete block formula of the form

$$A^{(0)}Y_m = Ey_n + hdf(y_n) + hbF(Y_m) \,, \quad (8)$$

$$Y_m = \begin{bmatrix} y_{n+1} & y_{n+\frac{3}{2}} & y_{n+2} & y_{n+\frac{5}{2}} & y_{n+3} & y_{n+4} & y_{n+5} \end{bmatrix}^T$$

$$y_n = \begin{bmatrix} y_{n-4} & y_{n-3} & y_{n-\frac{5}{2}} & y_{n-2} & y_{n-\frac{3}{2}} & y_{n-1} & y_n \end{bmatrix}^T$$

$$F(Y_m) = \begin{bmatrix} f_{n+1} & f_{n+\frac{3}{2}} & f_{n+2} & f_{n+\frac{5}{2}} & f_{n+3} & f_{n+4} & f_{n+5} \end{bmatrix}$$

$$f(y_n) = \begin{bmatrix} f_{n-4} & f_{n-3} & f_{n-\frac{5}{2}} & f_{n-2} & f_{n-\frac{3}{2}} & f_{n-1} & f_n \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \; E = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \; d = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{37529}{151200} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{7093}{28672} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{26}{105} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{191575}{774144} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1387}{5600} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{46}{189} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{185}{675} \end{bmatrix}$$

$$b = \begin{bmatrix} \frac{74437}{30240} & \frac{-4096}{945} & \frac{2713}{560} & \frac{-14864}{4725} & \frac{15011}{15120} & \frac{-2171}{30240} & \frac{27}{5600} \\ \frac{378927}{143360} & \frac{-623}{160} & \frac{333261}{71680} & \frac{-3411}{1120} & \frac{69151}{71680} & \frac{-2007}{28672} & \frac{135}{28672} \\ \frac{4969}{1890} & \frac{-128}{35} & \frac{523}{105} & \frac{-2944}{945} & \frac{103}{105} & \frac{-67}{945} & \frac{1}{210} \\ \frac{2040625}{774144} & \frac{-22375}{6048} & \frac{75625}{14336} & \frac{-17245}{6048} & \frac{370625}{387072} & \frac{-54125}{774144} & \frac{135}{28672} \\ \frac{2943}{1120} & \frac{-128}{35} & \frac{2889}{560} & \frac{-432}{175} & \frac{649}{560} & \frac{-81}{1120} & \frac{27}{5600} \\ \frac{2624}{945} & \frac{-4096}{945} & \frac{232}{35} & \frac{-4096}{945} & \frac{2624}{945} & \frac{46}{189} & 0 \\ \frac{10625}{6048} & 0 & \frac{-625}{336} & \frac{880}{189} & \frac{-625}{336} & \frac{10625}{6048} & \frac{185}{672} \end{bmatrix}$$

2.1 Analysis of basic properties of the self-starting block method

2.1.1 Order of the self-starting block method

Let the linear operator $L\{y(x);h\}$ associated with the block method (8) be defined as

$$L\{y(x);h\} = A^{(0)}Y_m - Ey_n - h(df(y_n) + bF(Y_m)) \tag{9}$$

Expanding using Taylor's series and comparing the coefficients of h gives

$$L\{y(x);h\} = c_0 y(x) + c_1 hy'(x) + c_2 h^2 y''(x) + \ldots + c_p h^p y^p(x) + c_{p+1} h^{p+1} y^{p+1}(x) + \ldots \tag{10}$$

Definition 2. *The linear operator* L *and the associated continuous linear multistep method (9) are said to be of order* p *if* $c_0 = c_1 = c_2 = \ldots = c_p = 0$ *and* $c_{p+1} \neq 0$ *is called the error constant and the truncation error is given by*

$$t_{n+k} = c_{p+1} h^{p+1} y^{p+1}(x_n) + 0(h^{p+2}) \tag{11}$$

For our method

$$L\{y(x);h\} = \left[\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n+1} \\ y_{n+\frac{3}{2}} \\ y_{n+2} \\ y_{n+\frac{5}{2}} \\ y_{n+3} \\ y_{n+4} \\ y_{n+5} \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{n-4} \\ y_{n-3} \\ y_{n-\frac{5}{2}} \\ y_{n-2} \\ y_{n-\frac{3}{2}} \\ y_{n-1} \\ y_n \end{bmatrix} \right.$$
$$\left. -h \begin{bmatrix} \frac{37529}{151200} & \frac{74437}{30240} & \frac{-4096}{945} & \frac{2713}{560} & \frac{-14864}{4725} & \frac{15011}{15120} & \frac{-2171}{30242} & \frac{27}{5600} \\ \frac{7093}{28672} & \frac{378927}{143360} & \frac{-623}{160} & \frac{333261}{71680} & \frac{-3411}{1120} & \frac{69151}{71680} & \frac{-2007}{28672} & \frac{135}{28672} \\ \frac{26}{105} & \frac{4969}{1890} & \frac{-128}{35} & \frac{523}{105} & \frac{-2944}{945} & \frac{103}{105} & \frac{-67}{945} & \frac{1}{210} \\ \frac{191575}{774144} & \frac{2040625}{774144} & \frac{-22375}{6048} & \frac{75625}{14336} & \frac{-17245}{6048} & \frac{370625}{387072} & \frac{-54125}{774144} & \frac{135}{28672} \\ \frac{1387}{5600} & \frac{2943}{1120} & \frac{-128}{35} & \frac{2889}{560} & \frac{-432}{175} & \frac{649}{560} & \frac{-81}{1120} & \frac{27}{5600} \\ \frac{46}{189} & \frac{2624}{945} & \frac{-4096}{945} & \frac{232}{35} & \frac{-4096}{945} & \frac{2624}{945} & \frac{46}{189} & 0 \\ \frac{185}{672} & \frac{10625}{6048} & 0 & \frac{-625}{336} & \frac{880}{189} & \frac{-625}{336} & \frac{10625}{6048} & \frac{185}{672} \end{bmatrix} \begin{bmatrix} f_n \\ f_{n+1} \\ f_{n+\frac{3}{2}} \\ f_{n+2} \\ f_{n+\frac{5}{2}} \\ f_{n+3} \\ f_{n+4} \\ f_{n+5} \end{bmatrix} \right]$$

Expanding in Taylor's series expansion gives and equating coefficients of the Taylor's series expansion to zero yield a constant order eight with the following error constants

$$c_0 = c_1 = c_2 = ... = c_8 = 0$$
$$c_9 = [-5.814(-04), 5.703(-04), 5.748(-04), -5.710(-04), -5.776(-04), -4.056(-04), -2.045(-03)]^T$$

2.2 Zero stability

Definition 3. *The block integrator (8) is said to be zero stable, if the roots* $z_s, s = 1, 2, ..., k$ *of the first characteristic polynomial* $\rho(z)$ *defined by* $\rho(z)\det(zA^{(0)} - E)$ *satisfies* $|z| \le 1$ *and every root satisfying* $|z| \le 1$ *have multiplicity not exceeding the order of the differential equation. Moreover, as* $h \to 0, \rho(z) = z^{r-\mu}(z-1)^{\mu}$ *where* μ *is the order of the differential equation,* r *is the order of the matrix* $A^{(0)}$ *and* E [11].

for our method

$$\rho(z) = \left[z\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \right] = 0$$

$$\rho(z) = z^6(z-1) = 0 \Rightarrow z_1 = z_2 = z_3 = z_4 = z_5 = z_6 = 0, z_7 = 1.$$

Hence, the self-starting block integrator is zero-stable.

2.3 Consistency

The block integrator (8) is consistent since it has order $\rho = 8 \ge 1$

2.4 Convergence

The self-starting block integrator is convergent by consequence of Dahlquist theorem below.

Theorem 4. *The necessary and sufficient conditions that a continuous LMM be convergent are that it be consistent and zero-stable [12].*

2.5 Region of absolute stability

Definition 5. Region of absolute stability is a region in the complex z plane, where $z = \lambda h$. It is defined as those values of z such that the numerical solutions of $y' = \lambda y$, satisfy $y_j \to 0 \; as \; j \to \infty$ for any initial condition.
The stability polynomial for our method is given by

$$= -h^7\left(\left(\frac{5}{448}\right)w^7 + \left(\frac{5}{192}\right)w^6\right) + h^6\left(\left(\frac{67}{1896}\right)w^7 - \left(\frac{1247}{8064}\right)w^6\right) - h^5\left(\left(\frac{1177}{3840}\right)w^7 + \left(\frac{711}{1280}\right)w^6\right)$$
$$- h^4\left(\left(\frac{451}{320}\right)w^6 - \left(\frac{853}{960}\right)w^7\right) - h^3\left(\left(\frac{355}{192}\right)w^7 + \left(\frac{165}{64}\right)w^6\right) + h^2\left(\left(\frac{85}{32}\right)w^7 - \left(\frac{105}{32}\right)w^6\right)$$
$$- h\left(\left(\frac{19}{8}\right)w^7 + \left(\frac{21}{8}\right)w^6\right) + w^7 - w^6$$

This gives the stability region shown in Fig. 1 shown below:

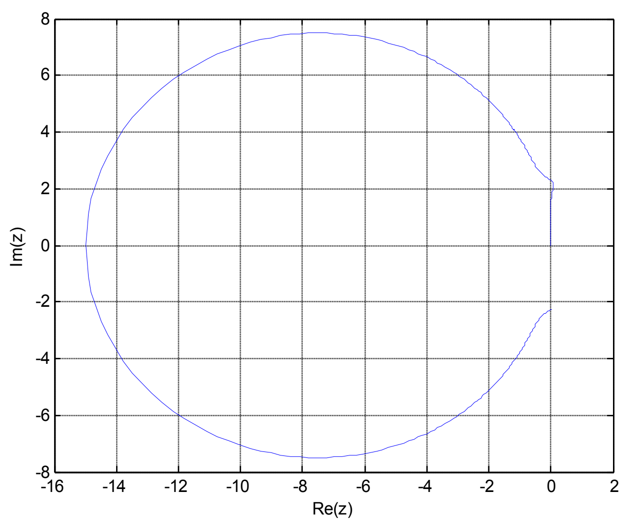

Fig. 1.

According to Fatunla, stiff algorithms have unbounded RAS, also Lambert showed that the stability region for L-stable schemes must encroach into the positive half of the complex plane.

3 Numerical Examples

We shall evaluate the performance of the self-starting block method on some challenging stiff problems and the results are displayed below.

The following notations are used in the tables below.

ERS - Error in Skwame et al. (2012)
ERM - Error in Mohammed and Yahaya (2012)

Problem 1. Consider the mildly stiff initial value problem

$$y' = -y,\ y(0) = 1,\ h = 0.1$$

With exact solution: $y(x) = e^{-x}$

This problem was solved by [8], by adopting fully implicit four points block backward formula.

Table 1. Showing the result for stiff problem 1

x	Exact Result	Computed Result	Error in our method	ERM
0.01	0.9048374180359595	0.9048374180555555	1.959599 (-011)	2.5292(-06)
0.02	0.8187307530779818	0.8187307555555556	2.477574 (-009)	2.0937(-06)
0.03	0.7408182206817179	0.7408182624999998	4.181828 (-008)	2.0079(-06)
0.04	0.6703200460356393	0.6703203555555557	3.095199 (-007)	1.6198(-06)
0.05	0.6065306597126334	0.6065321180555554	1.458543 (-006)	3.1608(-06)
0.06	0.5488116360940264	0.5488129556691562	1.319575 (-006)	2.7294(-06)
0.07	0.4965853037914095	0.4965864992843365	1.197493 (-006)	2.5457(-06)
0.08	0.4493289641172216	0.4493300698483614	1.105731 (-006)	2.1713(-06)
0.09	0.4065696597405992	0.4065708250308643	1.165290 (-006)	3.1008(-06)
0.10	0.3678794411714423	0.3678812102329587	1.769062 (-006)	2.7182(-06)

Problem 2. Consider the highly stiff initial value problem,

$$y' = -\lambda y, \, y(0) = 1, \, h = 0.01 \, , \, \lambda = 10$$

With exact solution: $y(x) = e^{-\lambda x}$

[9] solved this problem. by adopting an L-Stable hybrid block Simpson's method of order six.

Table 2. Showing the result for stiff problem 2

x	Exact Result	Computed Result	Error in our method	ERS
0.01	0.9048374180359595	0.9048374180555555	1.959599 (-011)	6.28(-03)
0.02	0.8187307530779818	0.8187307555555556	2.477574 (-009)	1.88(-03)
0.03	0.7408182206817179	0.7408182624999998	4.181828 (-008)	3.26(-03)
0.04	0.6703200460356393	0.6703203555555557	3.095199 (-007)	1.06(-02)
0.05	0.6065306597126334	0.6065321180555554	1.458543 (-006)	3.85(-03)
0.06	0.5488116360940264	0.5488129556691562	1.319575 (-006)	1.45(-03)
0.07	0.4965853037914095	0.4965864992843365	1.197493 (-006)	5.02(-04)
0.08	0.4493289641172216	0.4493300698483614	1.105731 (-006)	2.76(-04)
0.09	0.4065696597405992	0.4065708250308643	1.165290 (-006)	1.01(-04)
0.10	0.3678794411714423	0.3678812102329587	1.769062 (-006)	3.74(-05)

4 Discussion of the Results

We consider two numerical examples in this paper. Mohammed, U., and Yahaya, Y. A., (2010) solved the first problem (which is a mildly stiff), where they proposed a fully implicit four points block backward difference formula. Our method gave a better approximation because we proposed a self-starting method, which does not, required a starting value. Skwame, Y., Sunday, J., and Ibijola, E. A., (2012) solved the second problem. They adopted an L-Stable hybrid block Simpson's method of order six. Our method gave a better approximation, because the iteration per step in the new method is lower than the proposed method by Skwame, Y., Sunday, J., and Ibijola, E. A., (2012).

5 Conclusion

In this paper, we have presented a self-starting five-step eight-order block method for the solution of first order ordinary differential equations. The approximate solution adopted in this research produced a block method with L-stable stability region. This made it to perform well on stiff problems. The block method proposed was found to be zero-stable, consistent and convergent. The new block method was also found to perform better than the existing methods.

Competing Interests

Authors have declared that no competing interests exist.

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