Journal of Advances in Mathematics and Computer Science

26(4): 1-15, 2018; Article no.JAMCS.39949

ISSN: 2456-9968

(Past name: British Journal of Mathematics & Computer Science, Past ISSN: 2231-0851)

Modeling the Autocorrelated Errors in Time Series Regression: A Generalized Least Squares Approach

Emmanuel Alphonsus Akpan[1*] and Imoh Udo Moffat[2]

[1]Department of Mathematical Science, Abubakar Tafawa Balewa University, Bauchi, Nigeria.
[2]Department of Mathematics and Statistics, University of Uyo, Uyo, Nigeria.

Authors' contributions

This work was carried out in collaboration between both authors. Author EAA designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author IUM managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2018/39949
Editor(s):
(1) Feyzi Basar, Professor, Department of Mathematics, Fatih University, Turkey.
Reviewers:
(1) Yakubu Joy Asmau, Nigerian Defence Academy, Nigeria.
(2) E. H. Etuk, Rivers State University of Science and Technology, Nigeria.
(3) Razana Alwee, Universiti Teknologi Malaysia, Malaysia.
Complete Peer review History: http://www.sciencedomain.org/review-history/23379

Original Research Article

Received: 16th November 2017
Accepted: 22nd February 2018
Published: 28th February 2018

Abstract

This study considered Gross Domestic Product (N' Billion) as the dependent variable (denoted by Y_t), the Money Supply (N' Billion) as the independent variable (denoted by X_{1t}) and the Credit to Private Sector as another independent variable (denoted by X_{2t}). The data were obtained from the Central Bank of Nigeria Statistical Bulletin for a period ranging from 1981 to 2014. Each series consists of 34 observations. The study aimed at applying the generalized least squares to overcome the weaknesses of ordinary least squares to ensure the efficiency of the model parameters, unbiased standard errors, valid t-statistics and p-values, and to account for the presence of autocorrelation. Based on ordinary least squares fitted regression model, our findings revealed that X_{1t} and X_{2t} contributed significantly to Y_t and were able to explain about 67.95% of the variance in Y_t. However, the diagnosis of the fitted regression model using Breusch and Godfrey test, ACF, and PACF showed that the residuals are correlated, hence the need for generalized least squares. Further findings from the results of generalized least squares estimation revealed that their estimates are better and that the additional information in the error terms (autocorrelation) could be explained and captured by AR (2). Thus, it could be deduced that generalized least squares provides better estimates than the ordinary least squares and also accounts for autocorrelation in time series regression analysis.

**Corresponding author: E-mail: eubong44@gmail.com;*

Keywords: ARMA process; credit to private sector; GDP; money supply; Nigeria; ordinary least Squares; regression; time series.

1 Introduction

Classical regression model seeks to determine the relationship between the dependent variable and the independent variables. This regression model could be simple (consisting of one dependent and one independent variable) or multiple (consisting of one dependent and two or more independent variables). However, in the linear regression model, certain assumptions are made on how a dataset will be produced by an underlying data-generating process. According to [1], these assumptions include linearity (which ensures that the model specifies a linear relationship between the dependent and independent variables), homoscedasticity (which ensures that the error term has a finite constant variance), normality (which ensures that the error term is normally distributed) and no autocorrelation between the error terms (which ensures that the correlation in the error terms is zero). Moreover, regression model describes the value of the dependent variable as the sum of two parts, a deterministic part (explanatory variables) and the random part (error term).The error term is primarily a disturbance to an already stable relationship and is able to capture the remaining information in the dependent variable which could not be explained by the independent variables. Relating to the assumption on the error term, if the assumption of no correlation in the error term is violated, then, the underlying model would be rendered invalid with the standard errors of the parameters becoming biased. Moreover, if the errors are correlated, the least squares estimators are inefficient and the estimated variances are not appropriate [2-6]. By definition, autocorrelation is the lag correlation of a given series with itself, lagged by a number of time units (see [4]). Thus, when applying regression models to economic/management data in the presence of autocorrelation, the ordinary least squares estimation method ceases to provide efficient estimators and appropriate variances. In an attempt to overcome the weaknesses of ordinary least squares estimation method in the presence of autocorrelation, this study seeks to apply the generalized least squares estimation method since the least squares estimation method does not make use of the information of the unexplained variance as captured by the error terms in the dependent variable, whereas the generalized least squares (GLS) takes such information, the unexplained variance into account explicitly and is accomplished.

This study was motivated by the fact that some previous studies have failed to use GLS to explore the additional information embedded in the error terms of Ordinary Least Squares (OLS) estimated regression model involving Gross Domestic Product, Money Supply and Credit to Private Sector in Nigeria. For example, [7] investigated the impact of money supply on economic growth in Nigeria between 1980 and 2006 applying econometric technique ordinary least squares estimation, causality test and error correction models to time series data. The results revealed that although money supply is positively related to growth but the result is however insignificant in the case of gross domestic product growth rates on the choice between contractionary and expansionary money supply.

Bakare [8] examined the determinants of money supply growth and its implications on inflation in Nigeria. The study employed quasi-experimental research design approach for the data analysis. The results of the regression showed that credit expansion to the private sector determines money supply growth by the highest magnitude in Nigeria. The results also showed a positive relationship between money supply and inflation in Nigeria.

Babatunde and Shuaibu [9] studied the relationship between money supply, inflation and capital accumulation in Nigeria between 1970 and 2010. The study investigated the long run relationship between the variables using Johansen Cointegration test while error correction model was conducted on the variables to capture their short-run disequilibrium behaviour. Cointegration test results revealed that variables employed in the study shared long-run relationship. Also, the results of the error correction model indicated that money supply has a positive relationship to capital accumulation in Nigeria.

Chinaemerem and Chigbiu [10] investigated the impact of financial development variables on economic growth in Nigeria using Augmented Dickey-Fuller (ADF) test, Granger Causality test, Co-integration and Error Correction Method (ECM) were employed on time series data from 1960 – 2008. The results revealed that Money Supply (MS) and Credit to Private Sector (CPS) are positively related to the economic growth of Nigeria. The Johansen and Granger tests showed that Money Supply and Credit to Private Sector are co-integrated with GDP in Nigeria.

Inam [11] provided further evidence on the role of money supply on economic growth in Nigeria between 1985-2012 using augmented Cobb-Douglas production and relying on co-integration/Error correction methodology. It was found that money supply has a significant positive impact on economic growth in Nigeria.

Usman and Adejare [12] examined the effect of the money supply, foreign exchange on Nigeria economy using secondary data obtained from Central Bank of Nigeria Statistical bulletin covering the period of 1988 to 2010. Multiple regressions were employed in the data analysis. Narrow Money Supply, Broad Money, exchange rate and interest rate were found to have significant effects on the economic growth.

Yakubu and Affoi [13] analyzed the role of Commercial banks credit on economic growth in Nigeria from 1992 to 2012 using ordinary least squares. The findings revealed that Commercial bank credit has a significant effect on the economic growth in Nigeria.

Ujiju and Etale [14] examined the role of monetary policy instruments in controlling inflation in Nigeria using secondary time series panel data for the period covering 1982 to 2011. The study employed multiple regression technique and findings revealed that interest rate, minimum rediscount rate, liquidity ratio and cash reserve ratio had no significant influence on inflation.

Olowofeso et al. [15] examined the impacts of private sector credit on economic growth in Nigeria using the Gregory and Johansen co-integration test. The method was applied to quarterly data spanning 2000: Q1 to 2014: Q2, while the modified ordinary least squares procedure was employed to estimate the model coefficients. The study found a cointegrating relationship between output and its selected determinants, albeit, with a structural break in 2012: Q1. The error correction model confirmed a positive and statistically significant effect of private sector credit on output while increased prime lending rate inhibiting growth.

Solomon and Marshal [16] studied the linkage between finance companies intermediation functions and economic growth in Nigeria. Using an annual time series data spanning the period of 1992 – 2014 with the application of ordinary least squares, co-integration test and Granger causality test. The relative statistic results showed evidence for a strong and positive correlation between NLA and GDP in both short run and long run.

Nwoko et al. [17] examined the extent to which the Central Bank of Nigeria Monetary Policies could effectively be used to promote economic growth, covering the period of 1990-2011. The influence of money supply, average price, interest rate and labour force were tested on Gross Domestic Product using the multiple regression models as the main statistical tool for analysis. The findings indicate that average price and labour force have a significant influence on Gross Domestic Product while money supply was not significant. The interest rate was negative and statistically significant.

Inam and Ime [18] investigated the impact of monetary policy on the economic growth of Nigeria using annual data covering the period of 1970 to 2017 with the application of ordinary least squares technique and the Granger causality test. The results indicated a positive and insignificant relationship between money supply and economic growth. Also, no causality between money supply and economic growth was indicated.

2 Materials and Methods

2.1 Regression model

Rawlingset al. [3] defines a standard regression model as

$$Y_t = \beta_0 + \beta_{1,}X_{1,t} + \beta_2 X_{2,t} + \cdots + \beta_n X_{n,t} + \varepsilon_t \quad (1)$$

where

Y_t = dependent variable
β_i =regression parameters, i = 1,…, n
X_{it} =independent variables, i = 1,…, n
ε_t =error term assumed to be i.i.d. N(0, σ_t^2)

(see also [19, 20, 21, 5]).

Thus, the dependent variable for a time series regression model with independent variables is a linear combination of independent variables measured in the same time frame as the dependent variable. Estimates of the parameters of the model in (1) can be obtained by Least Squares Estimation Method (see more details in [22,3]).

2.2 Method of ordinary least squares for simple linear regression

The least squares estimation procedure uses the criterion that the solution must give the smallest possible sum of squared deviations of the observed Y_t from the estimates of their true means provided by the solution. Let $\hat{\beta}_o$ and $\hat{\beta}_1$ be numerical estimates of the parameters $\beta_0$and β_1, respectively, and let

$$\hat{Y}_t = \hat{\beta}_0 + \hat{\beta}_1 X_t \quad (2)$$

be the estimated mean of Y_t for each X_t, $t = 1, \dots$, n.

The least squares principle chooses $\hat{\beta}_0$ and $\hat{\beta}_1$ that minimize the sum of squares of the residuals, SSE:

$$\text{SSE} = \sum_{t=1}^{n} \left(Y_t - \hat{Y}_t\right)^2 = \sum_{t=1}^{n} \varepsilon_t^2 \quad (3)$$

where $\varepsilon_t = (Y_t - \hat{Y}_t)$ is the observed residual for the ith observation.

Also, we can express ε_i in terms of Y_t, X_t, β_0, and β_1. Hence, we have

$$\varepsilon_t = Y_t - \beta_0 - \beta_1 X_t \quad (4)$$

Equation (4) becomes

$$\text{SSE} = \sum_{t=1}^{n} (Y_t - \beta_0 - \beta_1 X_t)^2 \quad (5)$$

The partial derivative of SSE with respect to the regression constant, β_0, that is,

$$\frac{\partial SSE}{\partial \beta_0} = \frac{\partial}{\partial \beta_0}\left[\sum_{t=1}^{n} (Y_t - \beta_0 - \beta_1 X_t)^2\right] \quad (6)$$

with some subsequent rearrangement, the estimate of β_0 is obtained as

$$\hat{\beta}_0 = \left[\frac{\sum_{t=1}^{n} Y_t}{n}\right] - \beta_1 \left[\frac{\sum_{t=1}^{n} X_t}{n}\right] \quad (7)$$

The partial derivative of SSE with respect to the regression coefficient, β_1, that is,

$$\frac{\partial SSE}{\partial \beta_1} = \frac{\partial}{\partial \beta_1}[\sum_{t=1}^{n}(Y_t - \beta_0 - \beta_1 X_t)^2] \tag{8}$$

rearranging equation (8), we obtain the estimate of β_1

$$\hat{\beta}_1 = \frac{\sum_{t=1}^{n} Y_t X_t - \frac{\sum_{t=1}^{n} Y_t \sum_{t=1}^{n} X_t}{n}}{\sum_{t=1}^{n} X_t^2 - \frac{(\sum_{t=1}^{n} X_t)^2}{n}} \tag{9}$$

2.3 Method of generalized least squares (GLS)

Consider a simple regression model in (10)

$$Y_t = \beta_0 + \beta_1 X_t + \varepsilon_t \tag{10}$$

or put it in a matrix form

$$\mathrm{Y} = \mathrm{X}\beta + \varepsilon \tag{11}$$

where Y is the $n \times 1$ response vector: X is an $n \times k + 1$ model matrix: β is a $k + 1 \times 1$ vector of the regression coefficients to the estimate; and ε is an n×1 vector of errors. Assuming that $\varepsilon \sim N_n(0, \sigma^2 I_n)$ leads to well-known OLS estimator of β,

$$b_{OLS} = (X'X)^{-1}X'Y \tag{12}$$

with covariance matrix

$$\mathrm{Var}(b_{OLS}) = \sigma^2 (X'X)^{-1} \tag{13}$$

To generalize the OLS, we assume that $\varepsilon \sim N_n(0,\ \Sigma)$, where the error covariance matrix Σ is symmetric and positive-definite. The diagonal entries in Σ correspond to non-constant error variance, while nonzero off-diagonal entries correspond to correlated errors.

Given that Σ is known, the log-likelihood for the model is

$$log_e L(\beta) = -\frac{n}{2} log_e 2\pi - \frac{1}{2}\log_e(det\ \Sigma) - \frac{1}{2}(Y - \mathrm{X}\beta)'\Sigma^{-1}(Y - \mathrm{X}\beta) \tag{14}$$

which is minimized by GLS estimator of β,

$$b_{GLS} = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Y \tag{15}$$

with covariance matrix

$$\mathrm{Var}(b_{GLS}) = (X'\Sigma^{-1}X)^{-1} \tag{16}$$

Moreover, assuming that the process generating the regression error is stationary and the covariance of two errors depends only upon their separation (s) in time, it follows that:

$$\mathrm{Cov}(\varepsilon_t \varepsilon_{t+s} = Cov(\varepsilon_t \varepsilon_{t-s}) = \sigma^2 \rho_s \tag{17}$$

whereρ_s is the error autocorrelation at lag s.

Expressing$\sigma^2\rho_s$ in matrix form, we have:

$$\Sigma = \sigma^2 \begin{bmatrix} 1 & \rho_1 & \rho_2 & \cdots & \rho_{n-1} \\ \rho_1 & 1 & \rho_1 & \cdots & \rho_{n-2} \\ \rho_2 & \rho_1 & 1 & \cdots & \rho_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{n-1} & \rho_{n-2} & \rho_{n-3} & \cdots & 1 \end{bmatrix} = \sigma^2\rho \tag{18}$$

hence, for known values of σ^2 and ρ_s, then GLS estimator of β can be computed in a time series regression. In addition, in the error covariance matrix Σ, the large number (n-1) of different ρ_s makes their estimation impossible without specifying additional structure for the autocorrelated errors ([23]). Moreover, this additional could be specified to follow stationary time series models such as Autoregressive (AR), Moving Average (MA) and Autoregressive Moving Average (ARMA).

2.4 Autoregressive (AR) process

A stochastic process $\{R_t\}$ is an autoregressive process of order $p$and is denoted by $AR(p)$ ([24]), if

$$\hat{R}_t = \varphi_1\hat{R}_{t-1} + \varphi_2\hat{R}_{t-2} + \cdots + \varphi_p\hat{R}_{t-p} + a_t \tag{19}$$

where $\hat{R}_t = R_t - \mu$.

Using the backward shift operator, we have

$$\varphi(B)\hat{R}_t = a_t \tag{20}$$

where

(i) $B^s\hat{R}_t = \hat{R}_{t-s}$ is a backward shift operator
(ii) $\varphi(B) = 1 - \varphi_1 B - \cdots - \varphi_p B^p$
(iii) $\varphi_1, \varphi_2, \ldots, \varphi_p$ is a finite set of weighted parameters
(iv) $a_t$is a white noise process with mean, zero, and constant variance, σ^2.

Because $\sum_{j=1}^{p}|\varphi_j| < \infty$, is an important condition for invertibility of an autoregressive process of order p, then, the process is always invertible. For an autoregressive process to be stationary, the roots of $\varphi(B) = 1 - \varphi_1 B - \cdots - \varphi_p B^p = 0$, must lie outside of the unit circle.

The AR processes are useful in describing situations in which the present value of a time series depends on its preceding values plus a random shock. The covariance function can be obtained by multiplying $\hat{R}_{t-k}$ on both sides of (19) and taking expectations, we have

$$\gamma_k = \varphi_1\gamma_{k-1} + \varphi_2\gamma_{k-2} + \cdots + \varphi_p\gamma_{k-p}, \quad k > 0 \tag{21}$$

where $E(\hat{R}_{t-k}a_t) = \begin{cases} \sigma_a^2, & \text{for} k = 0, \\ 0, & \text{for} k > 0, \end{cases}$

Dividing (21) by γ_0, we have the following recursive relationship for the autocorrelation

$$\rho_k = \varphi_1\rho_{k-1} + \varphi_2\rho_{k-2} + \cdots + \varphi_p\rho_{k-p}, \quad k > 0. \tag{22}$$

2.5 Moving average (MA) process

According to [24], a stochastic process $\{R_t\}$ is a moving average process of order $q$and is denoted by $MA(q)$ if

$$\hat{R}_t = a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} \quad (23)$$

where$\hat{R}_t = R_t - \mu$.

Using the backward shift operator, we have

$$\hat{R}_t = \theta(B) a_t \quad (24)$$

where

(i) $B^s a_t = a_{t-s}$ is a backward shift operator
(ii) $\theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q$
(iii) $\theta_1, \theta_2, \dots, \theta_q$ is a finite set of weighted parameters
(iv) a_t is a white noise process with mean zero, and constant variance σ^2.

Because $1 + \theta_1^2 + \theta_2^2 + \cdots + \theta_q^2 < \infty$, is an important condition for stationarity of a moving average process of order q, thus, a finite moving average process is stationary. Moving average process is invertible if the root of $\theta(B) = 1 - \theta_1 B - \cdots - \theta_q B^q = 0$, lie outside of the unit circle. The moving average processes are useful in describing phenomena in which events produce an immediate effect that only lasts for short time periods.

For an $MA(q)$ process,

$$\hat{R}_t = a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} \quad (25)$$

$$\gamma_0 = E(R_t)^2 = E(R_t - \mu)^2 = E\left(a_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q}\right)^2$$

$$\gamma_0 = \sigma_a^2 + \theta_1^2 \sigma_a^2 + \cdots + \theta_q^2 \sigma_a^2$$

$$\gamma_0 = \sigma_a^2 \left(1 + \theta_1^2 + \cdots + \theta_q^2\right).$$

The variance is

$$\gamma_0 = \sigma_a^2 \sum_{j=0}^{q} \theta_j^2$$

where$\theta_0 = 1$ and$\sigma_a^2 = E(a_t^2)$

2.6 Autoregressive moving average (ARMA) process

A natural extension of pure autoregressive and pure moving average processes is the mixed autoregressive moving average ($ARMA$) processes, which includes the autoregressive and moving average as special cases [24].

A stochastic process $\{R_t\}$ is an $ARMA(p, q)$ process if $\{R_t\}$ is stationary and if for every t,

$$\varphi(B) \hat{R}_t = \theta(B) a_t \quad (26)$$

For the process to be invertible, we require that the roots of $\theta(B) = 0$, lie outside the unit circle. The stationary and invertible ARMA process can be written in pure autoregressive representation, that is,

$$\Pi(B)\hat{R}_t = a_t \quad (27)$$

$$\text{where}\,\Pi(B) = \frac{\varphi(B)}{\theta(B)} = (1 - \Pi_1 B - \Pi_2 B^2 - \cdots) \quad (28)$$

This process can also be written as pure moving average representation

$$\hat{R}_t = \psi(B)a_t \quad (29)$$

where$\psi(B) = \frac{\theta(B)}{\varphi(B)} = (1 - \psi_1 B - \psi_2 B^2 - \cdots)$

to derive the autocorrelation function, we rewrite

$$\hat{R}_t = \varphi_1\hat{R}_{t-1} + \varphi_2\hat{R}_{t-2} + \cdots + \varphi_p\hat{R}_{t-p} + a - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q}. \quad (30)$$

Multiplying (30) by $\hat{R}_{t-k}$ and taking the expectation, we have

$$\gamma_k = \varphi_1\gamma_{k-1} + \varphi_2\gamma_{k-2} + \cdots + \varphi_p\gamma_{k-p}, \quad k \geq q + 1 \quad (31)$$

where$E(\hat{R}_{t-k}a_{t-i}) = 0$ for $k > i$

dividing (31) by γ_0, we have the following recursive relationship for the autocorrelation

$$\rho_k = \varphi_1\rho_{k-1} + \varphi_2\rho_{k-2} + \cdots + \varphi_p\rho_{k-p}, \quad k \geq q + 1. \quad (32)$$

2.7 Detecting autocorrelation in the error terms

Breusch– Godfrey (BG) Test is used to detect the presence of autocorrelation in the residuals of a fitted regression model. For instance, assume that the error term ε_t in equation (10) follows the Pth–order autoregressive AR(P) process

$$\varepsilon_t = \rho_1\varepsilon_{t-1} + \rho_2\varepsilon_{t-2} + \cdots + \rho_p\varepsilon_{t-p} + v_t \quad (33)$$

where$v_t \sim N(0, \sigma^2)$

The null hypothesis H_0 to be tested is that

$H_0: \rho_1 = \rho_2 = \cdots = \rho_p = 0$

Then, the test statistic is (n - p)$R^2 \sim \chi^2_p$.

The decision rule is that if the calculated value of the BG test statistic exceeds the critical χ^2 value 5% level of significance (also, if the p-value corresponding to the BG test statistic is less than 0.05 level of significance), the hypothesis of no autocorrelation can be rejected; otherwise not rejected (see [25-27, 4]).

3 Results and Discussion

In this study, we consider Gross Domestic Product (N' Billion) as the dependent variable (denoted by Y_t), the Money Supply (N' Billion) as the independent variable (denoted by X_{1t}) and the Credit to Private Sector as another independent variable (denoted by X_{2t}). The data were obtained as available from the Central Bank of Nigeria Statistical Bulletin for a period ranging from 1981 to 2014. Each series consists of 34 observations.

Since our aim is to apply generalized least squares to overcome the weaknesses of ordinary least squares to ensure efficiency of model parameters and to explore the additional information embedded in the residuals of a fitted regression model, we begin by modelling the relationship between the dependent and independent variables through a linear regression. The estimated model is presented in equation (34) below:

$$Y_t = -410.8980 + 3.3476X_{1t} + 2.8239X_{2t} \tag{34}$$

s.e	(275.8978)	(0.4925)	(0.6868)
t-value	(-1.489)	(6.797)	(4.112)
p-value	(0.1439)	(2.837e-08)	(0.0002)

[Excerpts from Table 1]

Table 1. Output of regression Model

```
Call:
lm(formula = Y_t ~ X_1t + X_2t)

Residuals:
    Min      1Q  Median      3Q     Max
-2842.9 -1314.1   424.7   611.9  2686.0

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -410.8980   275.8978  -1.489 0.143877
X_1t           3.3476     0.4925   6.797 2.84e-08 ***
X_2t           2.8239     0.6868   4.112 0.000179 ***
---
Signif.codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1363 on 42 degrees of freedom
Multiple R-squared:  0.694,    Adjusted R-squared:  0.6795
F-statistic: 47.64 on 2 and 42 DF,  p-value: 1.58e-11
```

So far, from equation (34) we noticed that the two independent variables are significant since their p-value values are less than 5% significance level and were able to explain about 67.95% of the variance in Y_t. To diagnose the fitted regression model in equation (34), we plot the Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF) of the residuals of the model in equation (34).If the lags of the ACF and the PACF of the residuals of the fitted model are all zeros, then there is no additional information in the residual series and as such, the fitted model can be used for inference and future prediction. On the other hand, if the coefficient of the lag terms of both ACF and PACF are significant, then there remains additional information embedded in the residual series indicating the presence of autocorrelation. The implication is that, such a model is not efficient and cannot be used for inference; and in addition, the additional information can be modeled by an ARMA process.

Assessing the ACF and PACF in Figs. 1 and 2, we observed that the first 6 lags of the ACF and the first 2 lags of the PACF are significant. This shows that there remains additional information in the residual series and such information can be modelled by ARIMA process. Moreover, we applied Breusch-Godfrey test to further confirm the presence of serial correlation in the residual series. From Table 2, the Breusch-Godfrey test revealed that autocorrelation is present in the residual series since the Breusch-Godfrey test = 20.986 at lag1 with corresponding p-value = 4.626e-06 which is less than 5% level of significance (that is P = 4.626e-06< 0.05).

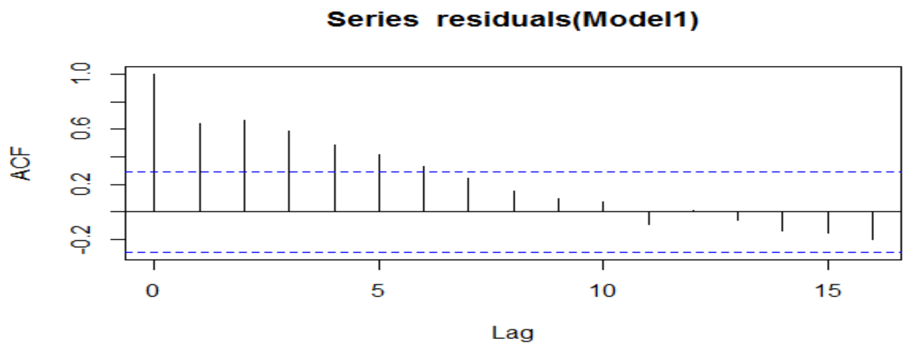

Fig. 1. ACF of the Residuals of Regression Model

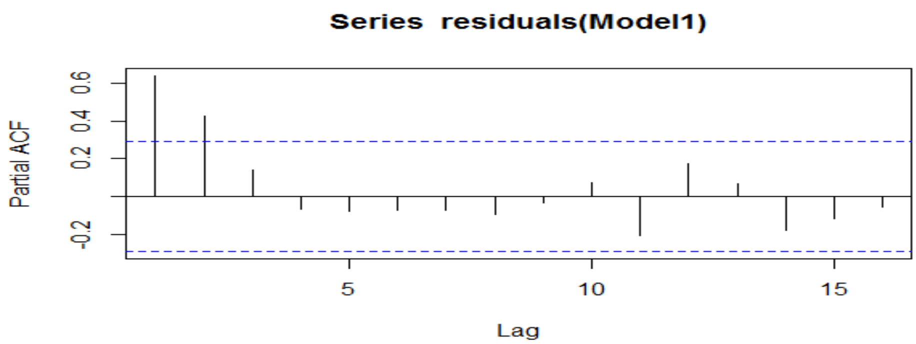

Fig. 2. PACF of the Residuals of Regression Model

Table 2. Breusch-Godfrey Test

Breusch-Godfrey test for serial correlation of order up to 1

data: Model1
LM test = 20.986, df = 1, p-value = 4.626e-06

Having detected and confirmed the presence of autocorrelation in the residual series, we moved to identify the order of Autoregressive Moving Average (ARMA) model that could capture the information in the autocorrelated errors. Looking at the ACF and PACF in Figs. 1 and 2, we observed that the ACF decays slowly to zero while there is a cut-off at lag 2 in PACF. This implies that the autoregressive component is of order 2 while the moving average component is of order zero. Hence, ARMA(2,0) or AR(2) model is identified.

Therefore, to account for the autocorrelation in the error terms, we entertained the Generalized Least Squares (GLS) model comprising both regression and Autoregressive (AR) equations. The regression component is presented in equation (35) while the AR component is presented in equation (36) below:

$$Y_t = -101.5704 + 2.8954X_{1t} + 2.4690X_{2t} \tag{35}$$

s.e	(767.1604)	(0.3788)	(0.4724)
t-value	(-0.1324)	(7.6434)	(5.2265)
p-value	(0.8953)	(0.0000)	(0.0000)

$$\varepsilon_t = 0.4209\varepsilon_{t-1} + 0.4242\varepsilon_{t-2} + v_t \tag{36}$$

[Excerpts from Table 3]

Table 3. Output of Generalized Least Squares Model

Generalized least squares fit by REML
Model: $Y_t \sim X_{1t} + X_{2t}$
Data: NULL

AIC	BIC	logLik
736.6719	747.0979	-362.3359

Correlation Structure: ARMA(2,0)
Formula: ~1
Parameter estimate(s):

Phi1	Phi2
0.4208681	0.4242165

Coefficients:

	Value	Std.Error	t-value	p-value
(Intercept)	-101.57037	767.1604	-0.132398	0.8953
X_{1t}	2.89541	0.3788	7.643414	0.0000
X_{2t}	2.46901	0.4724	5.226505	0.0000

Correlation:

	(Intr)	X_{1t}
X_{1t}	-0.105	
X_{2t}	-0.106	-0.118

Standardized residuals:

Min	Q1	Med	Q3	Max
-1.81821289	-0.89636900	0.08773882	0.27858563	1.99126950

Residual standard error: 1487.569
Degrees of freedom: 45 total; 42 residual

From equation (35), we observed that the two independent variables X_{1t} and X_{2t} are significant with their corresponding p-values less than 5% significance level, that is, (*P =0.00 <0.05*) and (*P = 0.00 < 0.05*) respectively. Also, it could be observed that a unit increase in X_{1t} increases Y_t by 2.8954 (N'Billion). Similarly, a unit increase in X_{2t} increases Y_t by 2.4690 (N'Billion). For the autocorrelated model in equation (36), the parameters of the AR model are significant given that we are 95% confident that 0.4209 (the coefficient of the first order term of the AR model) is between 0.1098 and 0.2351; and also, we are 95% confident that 0.4242 (the coefficient of the second order term of the AR model) is between -0.0683 and

0.7505 [as indicated in Table 4]. The implication is that, $X_{1t}$and X_{2t} significantly contributed to Y_t up to the last two years.

Table 4. Confidence Interval for Output of Generalized Least Squares

Approximate 95% confidence intervals

Coefficients:
lower est. upper
(Intercept) -1649.762804 -101.570366 1446.622072
X_{1t} 2.130937 2.895408 3.659880
X_{2t} 1.515666 2.469012 3.422358
attr(,"label")
[1] "Coefficients:"

Correlation structure:
lower est. upper
Phi1 0.10979522 0.4208681 0.2351397
Phi2 -0.06827319 0.4242165 0.7504677
attr(,"label")
[1] "Correlation structure:"

Residual standard error:
lower est. upper
772.639 1487.569 2864.029

Now, comparing the estimates of the ordinary least squares regression that does not account for autocorrelation with the estimates of generalized least squares regression that accounts for autocorrelation,

Table 5. Ordinary Least Squares (OLS) versus Generalized Least Squares (GLS)

Model	**Ordinary Least Squares (O L S)**	**Ordinary Least Squares (O L S)**	**Ordinary Least Squares (O L S)**	**Generalized Least Squares (GLS)**	**Generalized Least Squares (GLS)**	**Generalized Least Squares (GLS)**
	$\boldsymbol{\beta_{0[OLS]}}$	$\boldsymbol{\beta_{1[OLS]}}$	$\boldsymbol{\beta_{2[OLS]}}$	$\boldsymbol{\beta_{0[GLS]}}$	$\boldsymbol{\beta_{1[GLS]}}$	$\boldsymbol{\beta_{2[GLS]}}$
Parameter	-410.8980	3.3476	2.8239	-101.5704	2.8954	2.4690
Standard Error	275.8978	0.4925	0.6868	767.1604	0.3788	0.4724
t-value	-1.489	6.797	4.112	-0.1324	7.6434	5.2265
p-value	0.1439	2.837e-08	0.0002	0.8953	0.0000	0.00000

from Table 5, the main difference is the standard errors and the calculations based on the estimated variance of the coefficient probability distribution, that is, the coefficient of standard error, t-statistic and probability value (p-value). The standard errors are smaller except for that of the intercept when accounting for autocorrelation; that is to say, in GLS regression, the standard error, t-statistic and p-value are substantially different from those of the OLS regression. The implication is that GLS regression gives better estimates than the OLS regression.

4 Conclusion

This study explores the additional information (the unexplained variance that the independent variables could not capture) embedded in the error terms of the ordinary least squares estimated regression model and also ensures model efficiency. First of all, the relationship between the dependent variable, Y_t, and the independent variables, X_{1t} and X_{2t}, was determined using the ordinary least squares estimation method. The results of the ordinary least squares estimated regression revealed that X_{1t} and X_{2t} contributed significantly to Y_t and were able to explain about 67.95% of the variance in Y_t. Furthermore, evidence from Breusch and Godfrey test, ACF and PACF revealed that the error terms were autocorrelated. To address the autocorrelation in the error terms, we applied the generalized least squares and the results of our analysis revealed that the estimates of the regression model were better than those of the ordinary least squares. Also, the autocorrelation in the error terms was found to be completely modeled by AR(2) process. Therefore, our study showed that where the error terms of ordinary least squares estimated regression model are correlated, the model parameters become inefficient, the standard errors biased; and the t-statistics and the p-values no more valid. On the other hand, this study evidently proved that generalized least squares is a panacea for the weaknesses of ordinary least squares and accounted for the presence of autocorrelation in the error terms. Moreover, the findings of this study are in tandem with the study of [10] that money supply and credit to private sector have significant effects on the economic growth. Methodologically, this study differs from previous study by applying the generalized least squares and by modeling the additional information embedded in the error term through AR(2) process. Furthermore, it is recommended that this study be extended to cover the possible violation of assumption of the homoscedasticity.

Competing Interests

Authors have declared that no competing interests exist.

References

[1] Greene WH. Econometric analysis. 5th ed., New York: Prentice Hall. 2002;269.

[2] Granger CWJ, Newbold P. Spurious regressions in econometrics. Journal of Econometrics. 1974;2:111–120.

[3] Rawlings JO, Pantula SG, Dickey DA. Applied regression analysis: A research tool. 2nd ed., Springer; 1998.

[4] Gujarati DN. Basic econometrics. 4th ed. McGraw-Hill; 2004.

[5] Akpan EA, Moffat IU. Dynamic time series regression: A panacea for spurious correlations. International Journal of Scientific and Research Publications. 2016;6(10):337-342.

[6] Akpan EA, Moffat IU, Ekpo NB. Modeling regression with time series errors of gross domestic product on government expenditure. International Journal of Innovation and Applied Studies. 2016; 18(4): 990-996.

[7] Ogunmuyiwa MS, Ekone AF. Money supply – economic growth nexus in Nigeria. Journal of Social Sciences. 2010;22(3):199–204.

[8] Bakare AS. An empirical study of the determinants of money supply growth and its effects on inflation rate in Nigeria. Journal of Research in International Business and Management. 2011; 1(15):124-129.

[9] Babatunde MA, Shuaibu MI. Money supply, inflation and economic growth in Nigeria. Asian-African Journal of Economics and Econometrics. 2011;11(1):147-163.

[10] Chinaemerem OC, Chigbiu EE. An evaluation of financial development and economic growth of Nigeria: A causality test. Kuwait Chapter of Arabian Journal of Business and Management Review. 2012;1(10):22-44.

[11] Inam U. Money supply and economic growth in Nigeria: An econometric analysis. Journal of Economics and Sustainable Development. 2014;5(12):149-155.

[12] Usman OA, Adejare AT. Money supply, foreign exchange regimes and economic growth in Nigeria. Research Journal of Finance and Accounting. 2014;5(8):121-129.

[13] Yakubu Z, Affoi AY. An analysis of Commercial Bank's credit on economic growth in Nigeria. Current Research Journal of Economic Theory. 2014;6(2):11-15.

[14] Ujiju LE, Etale LM. Macroeconomic analysis of the relationship between monetary policy instruments and inflation in Nigeria. International Journal of Business and Management Review. 2016;4(6):31-39.

[15] Olowofeso EO, Adelek AO, Udoji AO. Impact of private sector credit on economic growth in Nigeria. CBN Journal of Applied Statistics. 2015;6(2):81-101.

[16] Solomon ID, Marshal I. The intermediation functions of finance companies and economic growth: Issues. Theory and Empirical Evidence from Nigeria. 2015;3(3):47-56.

[17] Nwoko NM, Ihemeje JC, Anumadu E. The impact of monetary policy on the economic growth of Nigeria. African Research Review. 2016;10(3):192-206.

[18] Inam US, Ime BS. Monetary policy and economic growth in Nigeria: Evidence from Nigeria. Advances in Social Sciences Research Journal. 2017;4(6):41-59.

[19] Pankratz A. Forecasting with dynamic regressions models. 3rded. New York, John Wiley and Sons; 1991.

[20] Dalgaard P. Introductory statistics with R. 2nd ed. Springer. 2008;228.

[21] Moffat IU, Akpan EA. Modeling and forecasting trend function of a discrete-time stochastic process. American Journal of Scientific and Industrial Research. 2014;56:195-202.

[22] Drasper NR, Smith H. Applied regression analysis. 3rded. New York, John Wiley and Sons; 1998.

[23] Fox J, Weisberg S. Time-series regression and generalized least squares in R: An appendix to an R companion to applied regression. 2nded; 2010.
Available:Citeseerx.ist.psu.edu›Viewdoc›download

[24] Wei WWS. Time series analysis univariate and multivariate methods. 2nd ed., AdisonWestley; 2006.

[25] Godfrey LG. Testing for higher order serial correlation in regression equations when the regressors include lagged dependent variables. Econometrica. 1978;46:1303-1310.
Available:http://dx.doi.org/10.2307/1913830.

[26] Breusch TS. Testing for autocorrelation in dynamic linear models. Australian Economic Papers. 1978;17:334–355.
Available:http://dx.doi.org/10.1111/j.1467 - 8454.1978.tb00635.x

[27] Breusch TS, Pagan AR. The Lagrange Multiplier Test and its Applications to Model Specification in Econometric s. Review of Economic Studies, 1980, 47: 239 – 253.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
http://www.sciencedomain.org/review-history/23379